VOLUME I

PERFORMANCE FLIGHT TESTING

ANNEX 6A

PROGRAMMED TEXT

FOR

SUPERSONIC AERODYNAMICS

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USAF TEST PILOT SCHOOL
EDWARDS AFB, CA

SUPERSONIC AERODYNAMICS

PROGRAMMED TEXT

INSTRUCTIONS

To use this study aid, fold a blank sheet of paper in half and place it on top of each page before you read anything on that page. Then slide the sheet down the page until you expose the first line of diagonals (///////). Now read the exposed information and question. Write your answer in the space provided, i.e., the letter preceding your choice in a multiple-choice or true-false question, the words which correctly complete a sentence containing blanks, or the numbers preceding your choices of answers which complete a "matching" question. More than one answer may be correct.

After writing your answer(s), slide the answer sheet down until you expose the correct answer(s) under the diagonals. If your answer is incorrect for a PRETEST question, stop for now and complete the TEACHING MATERIAL which follows that section.

If you miss a TEACHING MATERIAL question, review the preceding teaching material until you are satisfied you understand it. Review the teaching material at any time you need to, but don't waste time.

After checking your answer, slide the answer sheet down to the next diagonals and repeat the process above.

Note: The TEACHING MATERIAL is meant to supplement, not replace the materials in chapter 6 of the Performance text.

THERMODYNAMICS REVIEW

The aerodynamics and propulsion courses in the TPS syllabus make use of some of the basic relationships of thermodynamics. These review notes are designed to bring the student up to speed in some of the concepts of energy transfer that he will study later in the TPS course. It is of necessity brief and is designed to review the material, rather than to teach it from scratch.

The review material is divided into 4 topics as follows:

1. THE CONTINUUM - Defines the concept of continuous matter as distinguished from the kinetic theory or domain of molecular effects.

2. EQUATION OF STATE - Relates the properties of a gas at any given state.

3. LAW OF CONSERVATION OF MASS - Correlates the mass flow rates through a system.

4. NEWTON'S LAWS OF MOTION - Relates fluid motion with the forces causing it.

Amplifying material for any one of these sections can be found in any text on thermodynamics or gasdynamics. Hopefully, however, no outside text will be necessary.

1. THE CONTINUUM Any gas, such as air, is composed of a large number of molecules in continuous motion and collision. The most fundamental way of analyzing fluid motion would be write and solve the equations of motion for every particle. This approach could get out of hand in a hurry. Another possibility is to treat the motion of a large number of molecules on a statistical basis. This is the approach of the kinetic theory of gases or statistical mechanics. This theory has considerable merit, but is still too cumbersome for most practical calculations.

For the class of problems which we deal with the behavior of molecules is of little interest. We are concerned with the gross behavior of the fluid considered as continuous matter. Pressure, temperature and density are important to us; molecular speeds are not. It is valid to consider a fluid continuous when the smallest volume of fluid of interest to us contains so many molecules that average values are meaningful. Throughout these notes the parameters density, pressure, velocity, internal energy, enthalpy and entropy describe the gross behavior of a gas assumed to be continuous.

Density. If we consider the mass of gas Δm inside some volume ΔV we can discuss the average mass density as being the ratio $\Delta m/\Delta V$. If

ΔV is first taken as being rather large and then is subsequently shrunk about some point the value of $\Delta m/\Delta V$ would behave as shown in Figure 1.1.

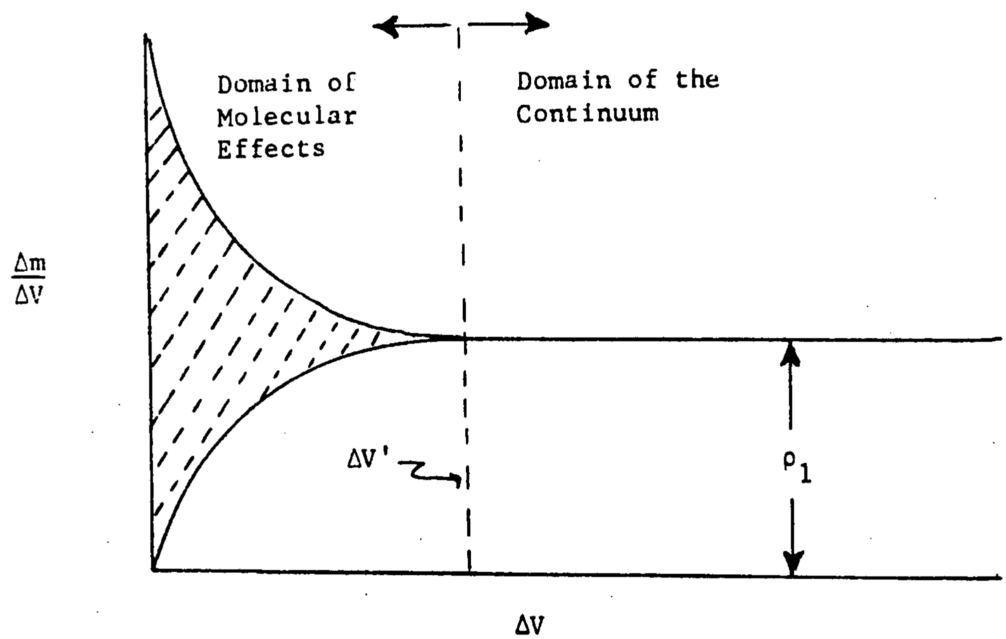


FIGURE 1.1

At first the mass density tends to be constant. As ΔV decreases, however, it reaches a point where it contains only a few molecules. In this area the density varies with time as molecules enter and leave the volume. The smallest volume which can be regarded as continuous then is $\Delta V'$ and we define the density <u>at a point</u> to be the density in some small volume $\Delta V'$ around the point. All other thermodynamic state characteristics are defined in similar fashion.

2. EQUATION OF STATE

There is a functional relationship between pressure, density and temperature at a point such that if any two of the parameters are known, the value of the third is fixed and can be calculated. This relationship changes somewhat with low temperatures and/or high pressures (in which case we must use Van der Waals' equation) but for the most part the Perfect Gas Law is appropriate. The Perfect Gas Law states

$$Pv = RT \quad , \qquad \text{OR} \tag{2-1}$$

$$P = \rho g RT \tag{2-2}$$

where

- P = absolute pressure in pounds per square foot
- ρ = mass density in slugs per cubic foot (slugs have the units of lb sec^2/ft)
- g = gravitational constant 32.2 ft/sec^2
- R = gas constant in units of foot-pounds per pound degree Rankine (for air $R = 53.3$)
- T = absolute temperature in degrees Rankine
- v = specific volume in cubic feet per pound mass.

Since the two equations are equivalent it can be seen that we have defined ρ as a mass density and v as a weight specific volume such that

$$\rho g = \frac{1}{v}$$

where

ρg then is weight density.

Some convenient numbers to remember are that P_{SL}, the standard day sea level pressure can be written as

$$P_{SL} = 14.7 \frac{Lb}{in^2} = 2116 \frac{Lb}{ft^2} = 29.92 \text{ in. Hg}$$

$$P_{gage} = P_{absolute} - P_{ambient}$$

The Rankine temperature scale is related to Fahrenheit as follows:

$$^{o}R = {}^{o}F + 460$$

Sea-level standard temperature is normally taken as $59^{o}F$ or $519^{o}R$.

Equivalent information for centegrade/Kelvin temperature scales is:

$$^{o}K = {}^{o}C + 273$$

Sea level standard temperature is $15^{o}C$ or $288^{o}K$.

NOTE: Another version of the Perfect Gas Law, not used in this text, is $P = \rho R_1 T$ where the value of R_1 differs from R by $R_1 = gR$. Always check units to be sure the equation matches the units of R.

In the engineering system of units R for air is 53.3 foot-pounds per pound degree Rankine. For gases other than air that obey the perfect gas law, R can be found by

$$R = \frac{1544}{\text{molecular weight}} \qquad \frac{\text{ft-lb}}{\text{lb}\ {}^{o}\text{R}}$$

This relationship comes simply from the empirical observation that for many gases the product of the gas constant and the molecular weight is a constant equal to 1544.

Example: What is the density of air at standard sea level conditions of 14.7 PSIA and 59^oF? Find both mass density and weight density.

$$Pv = RT$$

$$v = \frac{RT}{P} = \frac{(53.3\ \text{ft-lb/lb}^o\text{R})(460 + 59^o\text{R})}{(14.7\ \text{lb/in}^2)(144\ \text{in}^2/\text{ft}^2)} = 13.07\ \frac{\text{ft}^3}{\text{lb}}$$

$$\rho g = \frac{1}{v} = \frac{1}{13.07} = 0.0765 \text{ lb/ft}^3$$

$$\rho = \frac{1}{gv} = \frac{.0765}{32.2} = 0.002378 \text{ slugs/ft}^3$$

If we have a gas at some state 1 for which $P_1 v_1 = RT_1$ and we take the gas by some process to some state 2 for which $P_2 v_2 = RT_2$ then

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}\frac{v_2}{v_1}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}\frac{v_1}{v_2}$$

$$\frac{v_1}{v_2} = \frac{T_1}{T_2}\frac{P_2}{P_1}$$

These equations are nothing more than generalized statements of the laws of Boyle and Charles.

ADDITIONAL BUT ESSENTIAL DEFINITIONS

SPECIFIC HEAT

The specific heat of a substance is the amount of heat required to raise the temperature of one pound of mass by one degree Fahrenheit during some process. The unit of heat we use is the British Thermal Unit, which is defined as the amount of heat required to raise the temperature of one pound of water by one degree Fahrenheit. Logically enough, the specific heat of water is exactly 1.000 BTU/LB ^{O}R. For gases we must use two specific heats, however.

1. Specific heat at constant pressure, C_p (BTU/LB ^oR)

2. Specific heat at constant volume, C_v (BTU/LB ^oR).

The ratio C_p/C_v occurs frequently in practice and is defined as gamma (γ).

$$\gamma \equiv C_p/C_v \qquad (2\text{-}3)$$

The total heat required to change the temperature of an amount of substance is given by:

$$Q = WC\Delta T \qquad (2\text{-}4)$$

where

W = weight of substance in pounds

ΔT = temperature change $(T_2 - T_1)$ in ^oR

C = specific heat; C_p or C_v

SPEED OF SOUND

The speed of sound is the speed at which a small pressure pulse will propagate through a medium. The more compressible the fluid, the slower the speed of propagation as seen by

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

where a is acoustic velocity. This partial derivative can be evaluated for a perfect gas as

$$a = \sqrt{\gamma \, g \, RT} \qquad (2\text{-}5)$$

using the previously defined units for all the terms. For air, of course

$\gamma = 1.4$

$g = 32.2$ ft/sec^2

$R = 53.3$ ft-Lb/Lb $^\circ$R

$T = {}^\circ$Rankine

Two convenient conversion factors are:

$$a \text{ (knots)} = 29.1 \sqrt{T} \quad ({}^\circ\text{Rankine}) \qquad (2\text{-}6)$$

$$a \text{ (ft/sec)} = 49.1 \sqrt{T} \quad ({}^\circ\text{Rankine}) \qquad (2\text{-}7)$$

The speed of sound at sea level standard temperature is

$$a_{SL} = \sqrt{\gamma\, g\, R\, T_{SL}}$$

$$= \sqrt{1.4 \times 32.2 \times 53.3 \times 519}$$

$$= 1117 \text{ ft/sec} = 761 \text{ mph} = 661 \text{ knots}$$

MACH NUMBER

Ratio of the local velocity to the local speed of sound. We speak of flight Mach number as being the velocity of the aircraft compared to an acoustic velocity based on ambient temperature. It is possible to have local Mach numbers near the aircraft surface which vary significantly from the flight Mach number. For example, the local Mach number on the top surface of a wing reaches 1.0 at flight Mach numbers of 0.70 - 0.80.

$$M \equiv \frac{V}{a} = \frac{V}{\sqrt{\gamma\, g \cdot RT}} \tag{2-8}$$

Using Mach number as an index of the kind of flow we have, we can speak of 4 major flow regions: (The exact break points will vary, but those listed below show the approximate regions.)

$0 < M < .8$	subsonic
$.8 < M < 1.2$	transonic
$1.2 < M < 5$	supersonic
$5 < M < \infty$	hypersonic

Example:

a. What is the speed of sound at 35,000 feet at standard temperature? ($T = -65.8\ ^{\circ}F$)

$$a = \sqrt{\gamma\ g\ RT} = 49.1\sqrt{T} \quad \text{(in ft/sec)}$$

$$= 49.1\sqrt{460 - 65.8}$$

$$= 976 \text{ ft/sec}$$

b. At a true airspeed of 300 knots what is your Mach number at 35,000 feet? (1 knot = 1.69 ft/sec)

$$M = \frac{V}{a} = \frac{300 \times 1.69}{976} = .519$$

PROBLEMS

1. At what speed would you have to travel at 20,000 feet (standard temperature = 447.7 ^oR) to have a Mach number of 1.4?

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma g R T}}$$

$$= \frac{V}{49.1\sqrt{T}}$$

Alt = 20,000 ft.
T = 447.7° R

M = 1.4

$$1.4 = \frac{V}{49.1\sqrt{447.7}} \text{ ft/sec}$$

$$V = 1.4\,(49.1)\sqrt{447.7} \text{ ft/sec}$$

$$= 1.4\,(49.1)\,(21.15)$$

$$= 1454.46 \text{ ft/sec}$$

$$\boxed{V = 1454.5 \text{ ft/sec}}$$

2. What is the mass density (ρ) of air at a pressure of 10 pounds per square inch and a temperature of 400 ^oR?

Air $\gamma = 1.4$

P = 10 psi

T = 400°R

$$R = 53.3 \frac{\text{ft - lb}}{\text{lb °R}}$$

$$\text{Mass Density} = \rho = \frac{1}{gv}$$

$$V = \frac{RT}{P} = \frac{\left(53.3 \frac{\text{ft - lb}}{\text{lb} \cancel{°R}}\right) 400 \cancel{°R}}{10 \frac{\text{lb}}{\cancel{in^2}}\; 144 \frac{\cancel{in^2}}{ft^2}} = \frac{21320 \frac{\text{ft - }\cancel{\text{lb}}}{\text{lb}}}{1440 \frac{\cancel{\text{lb}}}{ft^2}}$$

$$= \underline{14.81 \ ft^3/lb}$$

$$\rho = \frac{1}{\left(32.2\ \text{ft}/\text{sec}^2\right)\left(14.81\ \text{ft}^3/\text{lb}\right)} = \frac{1}{476.88\ \frac{\text{ft}}{\text{lb. sec}^2}\ \text{ft}^3}$$

$$\boxed{\rho = .00210\ \frac{\text{slugs}}{\text{ft}^3}} \qquad = .00210\ \frac{\text{lb} - \text{sec}^2}{\text{ft}^4}$$

$$= .00210\ \text{slugs/ft}^3$$

3. A room (10 x 10 x 20 feet) contains standard sea-level air.

   (ρ = .002378 slugs/ft^3).

   a. What is the weight of the air?

   b. What is the mass of the air?

   c. What is the specific volume of the air?

$$\text{Room} = (10 \times 10 \times 20\ \text{ft}) = 2000\ \text{ft}^3$$

$$\rho = .002378\ \frac{\text{slugs}}{\text{ft}^3}$$

a. <u>Weight</u>

$$\rho = \frac{1}{gV}$$

$$\text{Weight Density} \quad \rho g = \frac{1}{V}\ \text{lb/ft}^3 = (.002378)\ \frac{\text{slugs}}{\text{ft}^3}\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)$$

$$\left(\text{Weight density} = \frac{\text{Weight}}{\text{Volume}}\right) = .07657\ \frac{\text{lb}}{\text{ft}^3}$$

$$\text{Weight} = \text{Weight density} \times \text{volume}$$

$$= \left(.07657\ \frac{\text{lb}}{\cancel{\text{ft}^3}}\right)\left(2000\ \cancel{\text{ft}^3}\right)$$

$$= 153.14\ \text{lb.}$$

b. Mass density $= \rho = \dfrac{\text{mass}}{\text{volume}}$

$$\text{Mass} = \rho \times \text{volume}$$

$$= .002378 \ \frac{\text{slugs}}{\cancel{ft^3}} \left(2000 \ \cancel{ft^3}\right)$$

$$= 4.756 \text{ slugs}$$

c. Specific volume $= \dfrac{\text{cubic ft}}{\text{lb mass}} = V$

$$\rho g = \frac{1}{V} = > V = \frac{1}{\rho g} = \frac{1}{.002378 \ \frac{\text{slugs}}{ft^3} \quad 32.2 \ \frac{ft}{sec^2}}$$

$$= \frac{1}{.07657 \ \frac{lb}{ft^3}} = \boxed{13.06 \ \frac{ft^3}{lb}}$$

4. How much heat must be added to the air in the room above to raise its temperature 1° Rankine? (C_v for air = 0.173 BTU/LB °R)

Raise temp 1°R $\longrightarrow \Delta T = 1°R$

Cv = 0.173 BTU/LB °R

$Q = WC\Delta T$

$T_2 - T_1 = 1°R$

$W = 153.14$ lb

$C_v = 0.173 \ \frac{BTU}{LB°R}$

$$Q = \left(153.14 \ \cancel{lb}\right) \left(0.173 \ \frac{BTU}{\cancel{LB°R}}\right) \left(1\cancel{°R}\right)$$

$$\boxed{Q = 26.49 \ BTU}$$

5. How much <u>additional</u> heat would have been required if the volume of room could expand to keep the pressure constant, rather than the volume? (C_p for air = 0.241).

$$Q = WC\Delta T \qquad C_p = 0.241$$

$$= \left(153.14\ \cancel{lb}\right)\left(0.241\ \frac{BTU}{\cancel{lb\,^\circ R}}\right)\left(1\,^\circ\cancel{R}\right)$$

$$= 36.91\ BTU$$

$$\text{Additional Heat Required} = \left(36.91 - 26.49\right)\ BTU$$

$$= \boxed{10.42\ BTU}$$

3. LAW OF CONSERVATION OF MASS

With the exception of nuclear reactions, matter can neither be created nor destroyed. This law can be stated as:

All Matter entering a System at the inlet	=	Matter leaving the System at the exit	+	Matter removed from the System	-	Matter added to to the System between entrance and exit

If we define G as a weight rate of flow (lb/sec) and $\dot{m}$ as a mass rate of flow (slugs/sec), then

$$G_{in} = G_{exit} + G_{removed} - G_{added} \quad \text{and}$$

$$\dot{m}_{in} = \dot{m}_{exit} + \dot{m}_{removed} - \dot{m}_{added}$$

by our definition $G = g\dot{m}$ where g is the gravitational constant.

G and $\dot{m}$ can be expressed as a product of density, cross section area, and velocity

$$\dot{m} = \rho AV \tag{3-1}$$

$$G = \rho g AV \tag{3-2}$$

where

ρ = mass density (slugs/ft^3)

ρg = weight density (pounds/ft^3)

A = cross sectional area (ft^2)

V = average flow velocity (ft/sec)

If we take the weight flow equation above and include the Perfect Gas Law and the definition of Mach number we can develop an expression for flow rate in terms of other parameters:

$$G = \rho \, g \, AV \qquad \text{and since}$$

$$P = \rho \, g \, RT, \qquad \therefore \qquad \rho g = \frac{P}{RT}$$

and substituting this value of ρg into the continuity equation

$$G = (\rho g) \, AV = \left(\frac{P}{RT}\right) AV$$

$$G = \frac{PAV}{RT}$$

We can obtain an expression for the RT term in the denominator from the definition of Mach number:

$$M \equiv \frac{V}{\sqrt{\gamma \, g \, RT}}$$

From before

$$G = PA \left(\frac{V}{RT}\right) = PA \left(\frac{V}{\sqrt{g \, \gamma \, RT}} \; \frac{\sqrt{\gamma \, g}}{\sqrt{RT}} \right)$$

$$\therefore \; G = PAM\sqrt{\frac{\gamma g}{RT}} \qquad (3\text{-}3)$$

There is nothing particularly significant about this form of the weight flow rate equation; it is included simply to show that the equation can be manipulated into other forms which might be more useful in a given problem.

There are a large number of flow problems in which no mass is added to or subtracted from the flow between the inlet and the exit. Typical

problems are flow in a streamtube, nozzle, diffuser, turbine or compressor. For these conditions:

$$G_{ent} = G_{exit} = \text{Constant}$$

$$\dot{m}_{ent} = \dot{m}_{exit} = \text{Constant}$$

$$\therefore \quad \rho AV = \text{Constant}$$

Furthermore for certain low velocity conditions the flow can be considered incompressible (ρ = Constant). For these conditions the continuity equation can be written

$$AV = \text{Constant}$$

Note that this is true <u>only</u> when the process can be considered incompressible.

EXAMPLE: Air at standard sea level conditions enters a nozzle with an inlet area of 40 square inches at a velocity of 20 feet/second. It leaves the nozzle through an area of 3.21 square inches at a pressure of 7 PSIA at 38^{o}F. What is the exit velocity and flow rate?

$$G_1 = \rho_1 \, g \, A_1 V_1 = \frac{P_1}{RT_1} A_1 V_1$$

$$= \frac{(14.7 \text{ lb/in}^2)}{(53.3 \text{ ft-lb/lb}^{o}\text{R})(519^{o}\text{R})} \times 40 \text{ in}^2 \times 20 \text{ ft/sec} = 0.425 \text{ lb/sec}$$

$$G_2 = G_1 = \frac{P_2}{RT_2} A_2 V_2 = 0.425 \text{ lb/sec}$$

$$V_2 = \frac{G_2 R T_2}{P_2 A_2}$$

$$= \frac{(.425\ \text{lb/sec})(53.3\ \text{ft-lb/lb}^\circ\text{R})(498^\circ\text{R})}{(7\ \text{lb/in}^2)(3.21\ \text{in}^2)}$$

$$= 502\ \text{ft/sec}$$

PROBLEMS

SECTION 3

6. Starting with the weight flow equation $G = \rho g AV$, the lecture notes derive the relationship $G = PAM\sqrt{\frac{\gamma g}{RT}}$. Starting with the mass flow equation, $\dot{m} = \rho AV$, show that the corresponding relationship is $\dot{m} = PAM\sqrt{\frac{\gamma}{g\ RT}}$.

$$G = \rho g AV \rightarrow G = PAM\sqrt{\frac{\gamma g}{RT}}$$

$$\dot{m} = \rho AV$$

show $\dot{m} = PAM\sqrt{\frac{\gamma}{gRT}}$

$$\dot{m} = \rho AV \qquad\qquad \rho g = \frac{P}{RT}$$

$$= \frac{P}{gRT} AV \qquad\qquad \rho = \frac{P}{gRT}$$

$$= PA\frac{V}{gRT} \qquad M = \frac{V}{\sqrt{\gamma gRT}} \quad \text{(pg 78)}$$

$$= \left(PA\frac{V}{\sqrt{\gamma gRT}} \quad \frac{\gamma}{\sqrt{gRT}}\right) \quad \text{Sub for M}$$

$$= PAM\frac{\sqrt{\gamma}}{\sqrt{gRT}}$$

$$\boxed{\dot{m} = PAM\sqrt{\frac{\gamma}{gRT}}}$$

7. Water flows through a 3 in^2 pipe at the rate of 1 lb per second. A nozzle at the end of the pipe has an exit area of 1/2 in^2. Calculate the exit velocity. (The density of water is 62.4 lb_f/ft^3).

$A_1 = 3\ in^2 \qquad A_2 = .5\ in^2 \qquad \rho g = 62.4\ lb_f/ft^3$

$= .021\ ft^2 \qquad = .0035\ ft^2$ (weight density)

$G_1 = 1\ \frac{lb_f}{sec} \qquad V_1 = ? \qquad V_2 = ?$

$G_1 = G_2$

$G_1 = \rho g A V \qquad \rho g =$ weight density

$$1\ \frac{lb_f}{sec} = 62.4_f\ \frac{lb}{ft^3}\ (.021\ ft^2)\ V_1$$

$$V_1 = \frac{1\ \cancel{lb}/sec}{\left(62.4\ \cancel{lb}/ft^{\cancel{3}}\right)\left(.021\ \cancel{ft^2}\right)}$$

$$= .763\ ft/sec$$

$$G_2 = \rho g A_2 V_2$$

$$V_2 = \frac{G_2}{\rho g A_2} = \frac{1\ \cancel{lb}/sec}{\left(62.4\ \frac{\cancel{lb}}{ft^{\cancel{3}}}\right)\left(.0035\ \cancel{ft^2}\right)}$$

$$= \frac{1}{.2184}\ \frac{ft}{sec}$$

$$\boxed{= 4.579\ \frac{ft}{sec}}$$

8. A constant area combustion chamber receives air and fuel at its entrance. The mixture is at 200°F, 75 PSIA, and has a velocity of 50 feet per second. At exit the temperature is 1500°F and the pressure is 65 PSIA. What is the exit velocity and Mach number? ($R = 53.3$, $\gamma = 1.4$).

$$A = \text{Constant}$$

$$T_1 = 200°F \qquad T_2 = 1500°F \qquad R = 53.3 \ \frac{\text{FtLb}}{\text{lb°R}}$$

$$= 660°R \qquad = 1960°R$$

$$°R = °F + 460 \qquad \gamma = 1.4$$

$$P_1 = 75 \ \frac{\text{lb}}{\text{in}^2} \qquad P_2 = 65 \ \frac{\text{lb}}{\text{in}^2}$$

$$V_1 = 50 \ \frac{\text{ft}}{\text{sec}} \qquad V_2 = ? \qquad M = ?$$

$$G_1 = \frac{PAV}{RT} = \frac{\left(75 \frac{\text{lb}}{\text{in}}\right)(A)(50 \text{ ft/sec})}{\left(53.3 \ \frac{\text{ft-lb}}{\text{lb°R}}\right)\left(660°R\right)} = .1066\, A_1 \ (\text{lb/sec})$$

$$G_2 = G_1 = \frac{P_2}{RT_2} A_2 V_2$$

$$\rightarrow V_2 = \frac{G_1 R T_2}{P_2 A_2} = \frac{(.1066 A_1)(53.3)(1960)}{65 \frac{\text{lb}}{\text{in}^2} A_2}$$

$$A_2 = A_1$$

$$\boxed{V_2 = 171.33 \ \text{ft/sec}}$$

$$M = \frac{V}{\sqrt{\gamma g R T}}$$

$$= \frac{171.33}{\sqrt{(1.4)(32.2)(53.3)(1960)}} = \boxed{.0789}$$

9. **A diffuser which has an entrance area of 1.5 ft^2 is operated with an entrance velocity of 500 knots in standard conditions at 20,000 feet pressure altitude. What is the weight flow in pounds per second? (1 kt = 1.69 ft/sec $\rho(20{,}000)$ = 0.001267 slugs/ft^3).**

$A_{int} = 1.5\ ft^2$

$V_{int} = 500\ kts$

Press Alt = 20000 ft $\qquad \rho(20000) = 0.001267\ \dfrac{slugs}{ft^3}$

Weight Flow = G = ?

$$V_{int} = 500\ \cancel{kts}\left(1.69\ \frac{ft/sec}{\cancel{kt}}\right) = 845\ \frac{ft}{sec}$$

$$G = \rho g A V$$

$$= \left(.001267\ \frac{lb_f\text{-}\cancel{sec^2}}{\cancel{ft^4}}\right)\left(32.2\ \frac{\cancel{ft}}{\cancel{sec^2}}\right)\left(1.5\ \cancel{ft^2}\right)\left(845\ \frac{\cancel{ft}}{sec}\right)$$

$$= \boxed{51.71\ \frac{lb_f}{sec}}$$

10. The exhaust gases (molecular weight = 29.90) of a turbojet engine expand through a nozzle of 1 square foot exit area. If the exhaust gas temperature is 1500 °R, the pressure is 20 PSIA, and the Mach number is 1.0, what is the weight flow in pounds per second? ($\gamma = 1.40$)

$$\text{molecular wt} = 29.90 \Rightarrow R = \frac{1544}{29.90} \frac{\text{ft - lb}}{\text{lb°R}}$$

$$A_{exit} = 1 \text{ ft}^2 \qquad = 51.64 \frac{\text{ft - lb}}{\text{Lb°R}}$$

$$T_2 = 1500°R$$

$$P_2 = 20 \frac{\text{lb}}{\text{in}^2}$$

$$M = 1.0 \qquad \gamma = 1.4$$

$$G = PAM\sqrt{\frac{\gamma g}{RT}} = \left(20 \frac{\text{lb}}{\cancel{\text{in}^2}}\right)\left(\frac{144 \cancel{\text{in}^2}}{\cancel{\text{ft}^2}}\right)\left(1 \cancel{\text{ft}^2}\right)\left(1.0\right)\sqrt{\frac{(1.4)\left(32.2 \ \cancel{\text{ft}}/\text{sec}^2\right)}{\left(51.64 \frac{\cancel{\text{ft lb}}}{\cancel{\text{lb°R}}}\right)\left(1500\cancel{°R}\right)}}$$

$$= \left(2880 \text{ lb}\right)\left(\frac{.02412}{\text{sec}}\right) = 69.478 \frac{\text{lb}}{\text{sec}}$$

$$= \boxed{69.478 \text{ lb/sec}}$$

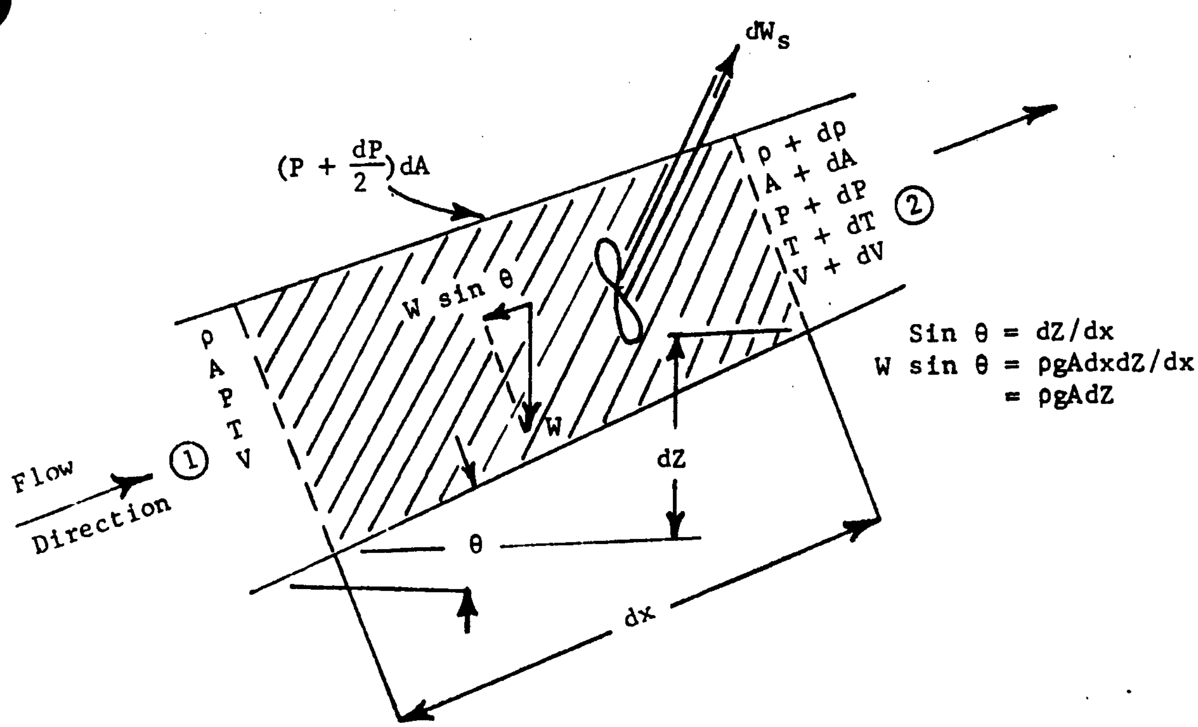
4. NEWTON'S LAWS OF MOTION

1. A particle at rest or traveling at constant velocity will tend to remain at rest or at a constant velocity.

2. A particle acted upon by an unbalanced force will accelerate. The acceleration will be directly proportional to the unbalanced force and inversely proportional to the mass of the particle. $F = ma$

3. Every action has an equal and opposite reaction.

These laws can be applied to a volume of fluid as well as a particle. Let's consider Newton's law applied to flow through some passage. In practice this passage could be a streamtube around an airfoil, flow through a nozzle, a diffuser, or any other machine.



If equations are developed for one-dimensional flow and we consider the forces which act on this chunk of matter and act along the axis of flow we obtain:

a. Pressure forces acting on the front face (PA), on the rear face $(P + dP)(A + dA)$, and on the sides $(P + \frac{dP}{2})dA$.

b. Weight forces ($W \sin \theta = \rho\, gAdZ$) acting at the center of gravity.

c. Shaft of shear forces set up in a shaft which adds work to, or removes work from, a system (dW_s).

d. Friction forces along the side face (dW_f).

$$\overbrace{P_A - (P + dP)(A + dA) + (P + \frac{dP}{2})\, dA}^{\text{Pressure Forces}} - \overbrace{\rho g\; Adz}^{\text{weight forces}} = ma - dw_s - dw_f$$

After much manipulation and

1) neglecting higher order terms

2) friction

3) change in potential energy

4) neglecting shaft work done

5) integration from 1 state to another

6) assuming an isentropic process ($P_V{}^{\gamma} = C$) we obtain Bernoulli's Compressible Flow equation

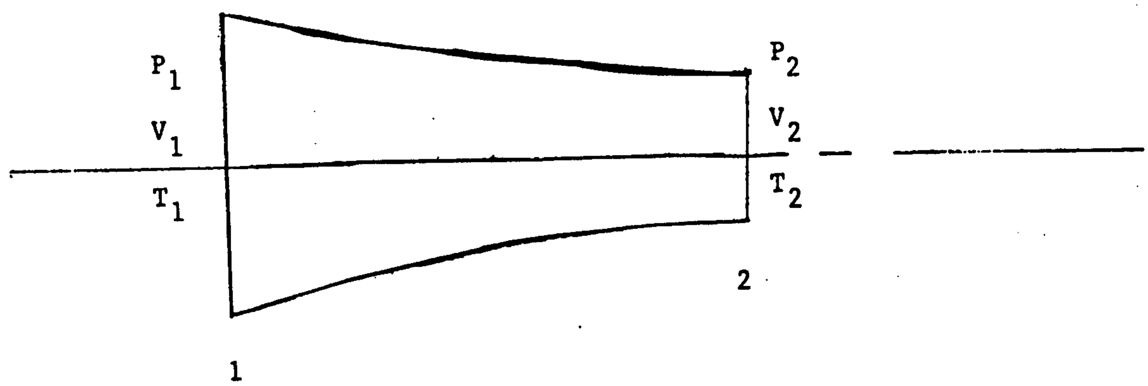
$$\frac{V_2{}^2 - V_1{}^2}{2g} + \frac{\gamma}{\gamma - 1}\left[\frac{P_2}{\rho_2 g} - \frac{P_1}{\rho_1 g}\right] = o$$

If we then assume that density is constant, we obtain Bernoulli's Incompressible Flow equation

$$\frac{V_2{}^2 - V_1{}^2}{2g} + \frac{1}{\rho g}(P_2 - P_1) = o$$

Bernoulli's equations apply to flow processes where there is neither friction nor shaft work and the flow is either (1) incompressible, or (2) isentropic.

Application of Bernoulli Equations to Flow through a nozzle.



The Bernoulli equations now permit us to calculate fluid velocity at any point knowing only the pressure and density at the point in question and some reference state.

EXAMPLE: Air (compressible fluid, $\gamma = 1.4$) flows through a nozzle under the following conditions:

$$P_1 = 20 \text{ PSIA}$$

$$T_1 = 1600^{\circ}\text{F}$$

$$V_1 = 0$$

$$\rho_1 = \frac{P_1}{gRT_1} = \frac{(20 \times 144)}{(32.2)(53.3)(1600 + 460)} = 0.000816 \ \frac{\text{slugs}}{\text{FT}^3}$$

$$P_2 = 14.7 \text{ PSIA}$$

$$T_2 = 1405^{\circ}\text{F}$$

$$\rho_2 = \frac{P_2}{gRT_2} = \frac{(14.7 \times 144)}{(32.2)(53.3)(1405 + 460)} = 0.00066 \ \frac{\text{slugs}}{\text{FT}^3}$$

$$V_2 = ?$$

State 1 is the reference condition. The velocity at state 2 is the unknown.

If $$\frac{V_2^2 - V_1^2}{2g} + \frac{\gamma}{\gamma - 1} \left[\frac{P_2}{\rho_2 g} - \frac{P_1}{\rho_1 g}\right] = 0$$

then

$$V_2^2 = V_1^2 + \frac{2\gamma}{\gamma - 1} \left[\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right]$$

$$= 0 + \frac{2 \times 1.4}{1.4 - 1} \left[\frac{20 \times 144}{.000814} - \frac{14.7 \times 144}{.00066}\right]$$

$$= 2.32 \times 10^6$$

$$\therefore V_2 = 1520 \text{ Ft/sec}$$

THRUST EQUATION

We can use Newton's Second Law to calculate the thrust from a jet engine, ramjet, rocket, propeller, or any other reaction device. Considering steady flow:

$$F = ma$$

$$= (\dot{m}\,\Delta T)\left(\frac{\Delta V}{\Delta T}\right)$$

$$= \dot{m}\,\Delta V \qquad (4\text{-}6)$$

11. The Bernoulli equation for incompressible flow was shown in the notes to be $\frac{V_2^2 - V_1^2}{2g} + \frac{1}{\rho g}(P_2 - P_1) = 0.$

Show that this can be rearranged into the form

$$P + \frac{1}{2}\rho V^2 = \text{Constant}$$

$$\frac{V_2^{\ 2} - V_1^{\ 2}}{2g} + \frac{1}{\rho g}\left(P_2 - P_1\right) = 0 \qquad \underline{\text{Incompressible Flow}}$$

$$\frac{V_2^{\ 2} - V_1^{\ 2}}{2g} + \frac{1}{\rho g}\left(P_2 - P_1\right) = 0$$

$$\frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} + \frac{P_2}{\rho g} - \frac{P_1}{\rho g} = 0$$

$$\frac{V_2^{\ 2}}{2g} + \frac{P_2}{\rho g} = \frac{V_1^{\ 2}}{2g} + \frac{P_1}{\rho g}$$

$$\rho V_2^{\ 2} + 2P_2 = \rho V_1^{\ 2} + 2P_1$$

$$P_2 + \frac{1}{2}\rho V_2^{\ 2} = P_1 + \frac{1}{2}\rho V_1^{\ 2} \rightarrow \text{Constant}$$

$$P + \frac{1}{2}\rho V^2 = \text{Constant}$$

12. In similar fashion, show that compressible Bernoulli,

$$\frac{V_2^2 - V_1^2}{2g} + \frac{\gamma}{\gamma - 1}\left(\frac{P_2}{\rho_2 g} - \frac{P_1}{\rho_1 g}\right) = 0 \quad \text{can be}$$

rearranged to read

$$\frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\left(\frac{P}{\rho}\right) = \text{Constant}$$

$$\frac{V_2^{\ 2} - V_1^{\ 2}}{2\ g} + \frac{\gamma}{\gamma - 1}\left(\frac{P_2}{\rho_2 g} - \frac{P_1}{\rho_1 g}\right) = 0 \qquad \underline{\text{Compressible}}$$

$$\frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} + \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P_2}{\rho_2 g}\right) - \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P_1}{\rho_1 g}\right) = 0$$

$$\frac{V_2^{\ 2}}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P_2}{\rho_2}\right) = \frac{V_1^{\ 2}}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P_1}{\rho_1}\right) \Rightarrow \text{Constant}$$

$$\frac{V^{\ 2}}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P}{\rho}\right) = \text{Constant}$$

13. A jet engine which is operated at velocity of 400 ft/sec consumes air at the rate of 32.2 lb/sec and discharges the air at a velocity of 1200 ft/sec.

a. What is the force required to change the momentum of the air?

b. What thrust is developed by the engine?

c. What is the direction of these two forces?

$$V = 400 \frac{ft}{sec} \qquad G = 32.2 \frac{Lb}{sec}$$

$$V_{exchange} = 1200 \text{ ft/sec}$$

a. $F = Ma = \dot{M}\Delta V$ Forces required to change momentum of the air

$$= \frac{G}{g} \Delta V$$

$$= \frac{32.2 \ \frac{lb}{sec}}{32.2 \ ft/_{sec^2}} \left(1200 \frac{ft}{sec} - 400 \frac{ft}{sec}\right)$$

$$= \frac{32.2}{32.2} \frac{lb}{ft/_{sec}} \left(800 \frac{ft}{sec}\right)$$

$$= \boxed{800 \text{ Lb}}$$

b. $F = \frac{G}{g} \Delta V$ Thrust Equ

from (a) Thrust developed = $\boxed{800 \text{ lb.}}$

c. Air → [Engine] ← Thrust

Force (momentum)

Engine

Force → Thrust ←

$\boxed{\text{Forces act in opposite directions}}$

14. Standard sea level air ($P = 14.7$ PSIA, $\rho = 0.00238 \frac{slugs}{ft^3}$, $T = 519^oR$) enters a converging duct at essentially zero speed. It exits the duct at $P = 7.76$ PSIA, $\rho = 0.001508 \frac{slugs}{ft}$. Find the exit velocity and Mach number.

$$P_1 = 14.7 \frac{Lb}{in^2} \qquad \rho_1 = 0.00238 \frac{slugs}{ft^3}$$

$$= 14.7 \frac{Lb}{\cancel{in^2}} \left(144 \frac{\cancel{in^2}}{ft^2}\right) \qquad T_i = 519°R$$

$$= 2116.8 \frac{Lb}{ft^2} \qquad V_1 = 0$$

$$P_2 = 7.76 \frac{Lb}{in^2} \qquad V_2 = ?$$

$$= 7.76 \frac{Lb}{\cancel{in^2}} \left(144 \frac{\cancel{in^2}}{ft^2}\right) \qquad \rho2 = 0.001508 \frac{Slugs}{ft^3}$$

$$= 1117.44 \frac{Lb}{ft^2}$$

Air is compressible $\gamma = 1.4$

$$\frac{V_2^2}{2} + \frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2}\right) = \cancel{\frac{V_1^2}{2}}^{\,0} + \frac{\gamma}{\gamma-1}\left(\frac{P_1}{\rho_1}\right)$$

$$\frac{V_2^2}{2} = \frac{\gamma}{\gamma-1}\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)$$

$$V_2^2 = 2\left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)$$

$$= 2\left(\frac{1.4}{1.4-1}\right)\left(\frac{2116.8 \frac{Lb}{\cancel{ft^2}}}{.00238 \frac{Lb\ Sec^2}{ft^{\cancel{4}\,2}}} - \frac{1117.44 \ Lb/\cancel{ft^2}}{.001508 \frac{\cancel{Lb}\text{-}sec^2}{ft^{\cancel{4}\,2}}}\right)$$

$$V_2^2 = 2\,(3.5)\left(889411.765\ \frac{ft^2}{sec^2} - 741007.958\ \frac{ft^2}{sec^2}\right)$$

$$= 7\left(148403.807\ \frac{ft^2}{sec^2}\right) = 1.0388 \times 10^6$$

$$V = \sqrt{1.0388 \times 10^6\ \frac{ft^2}{sec^2}}$$

$$= \boxed{1019.23\ \frac{ft}{sec}}$$

15. The German V-2 rocket engine consumed propellants at about 270 lb per second and the exit gas velocity was found to be 6000 feet per second.

a. What is the thrust when the velocity is zero?

b. What is the thrust when the rocket velocity is 6000 feet per second?

c. What is the thrust horsepower when the rocket velocity is 6000 feet per second?

(Thrust horsepower = Thrust (pounds) X velocity (feet per sec) ÷ 550).

$G = 270\ \frac{lb}{sec}$ $\qquad V_2 = 6000\ \frac{ft}{sec}$

a. thrust when velocity = 0 i.e. $V_1 = 0$

Rocket velocity = 0

$$F = \frac{G}{g}\,\Delta V \quad \text{(Thrust Eqn)}$$

$$= \frac{270\ lb/sec}{32.2\ ft/sec^2}\left(6000\ \frac{ft}{sec} - 0\right)$$

$$= \boxed{50310.56\ lb}$$

b. Rocket vel. = 6000 ft/sec $\quad V_1 = 0$ Since all fuel consumed is on Rocket

$$F = \frac{G}{g}\,\Delta V \quad \text{Thrust Eqn}$$

$$= \frac{270 \text{ lb/sec}}{32.2 \ \frac{\text{ft}}{\text{sec}^2}} (6000 - 0) = \boxed{50310.56 \text{ lb}}$$

c. Thrust Horsepower

Rocket vel. is 6000 $\frac{\text{ft}}{\text{sec}}$

$$T_{Hp} = \frac{\left[T(\text{Lb})\right] \left[\text{Velocity}\left(\frac{\text{ft}}{\text{sec}}\right)\right]}{550}$$

$$= \frac{(50310.56 \text{ lb}) \ (6000 \text{ ft/sec})}{550 \ \frac{\text{ft-lb}}{\text{sec Hp}}}$$

$$= \boxed{548{,}842.46 \ \text{Hp}}$$

UNIT ONE

INTRODUCTION

PRETEST

Basic Assumptions and Equations

1. The fluid used for supersonic aerodynamic analysis is:

   a. Homogeneous (properties are not a function of position)

   b. Nonhomogeneous

   c. Compressible ($\frac{dv}{dp} \neq 0$)

   d. Incompressible

   e. Viscous ($\mu = 0$, outside boundary layer)

   f. Non-Viscous

   g. Elastic (sound wave can propogate)

   h. In-elastic

////////////////////////////////

a, c, f, g

2. To develop the basic equations we assume the control volume has the following properties:

   a. Properties are a function of time

   b. Adiabatic (dq across the control volume = 0)

   c. Isentropic

   d. One-dimensional (radius of curvature and $\frac{dA}{1}$ are negligible)

////////////////////////////////

b, d

3. The speed of sound is a function of:

   a. Pressure

   b. Temperature

   c. Density

   d. $49\sqrt{T}$ ft/sec (for air)

///////////////////////////////

b, d

4. Mach number is always based on velocity and the local speed of sound.

   a. True

   b. False

///////////////////////////////

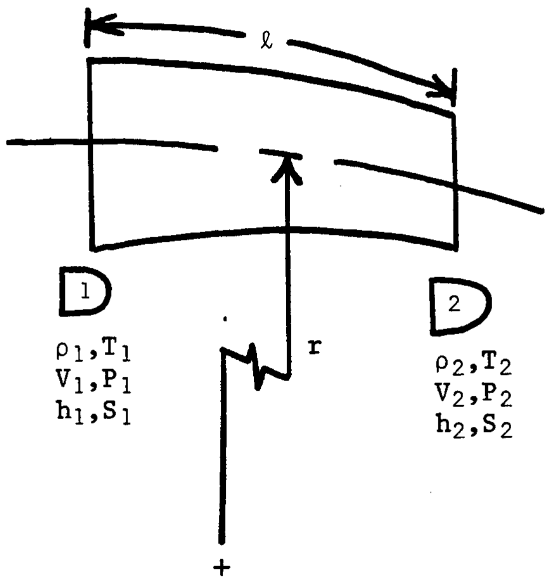
b. False , it can be based on speed of sound based on some other temp.

If you correctly answered the above questions, you have completed Unit 1. Go to page 2-1.

If you think you need more instruction, complete the following pages.

A real gas is a compressible, viscous, elastic, nonhomogenous and chemically reactive fluid. However, it has been discovered that the use of idealized fluids will provide acceptable solutions to fluid dynamic problems. For problems involving supersonic flow (below Mach 5) outside of the boundary layer reasonable results are gained by dealing with a fluid that is homogeneous, non-viscous, compressible and elastic.

The flow is defined by a control volume and equations are developed which relate the values of properties at one station to another.



This control volume is assumed to be steady state, i.e. $\frac{d}{dt} = 0$, the heat flow is zero across the boundaries (adiabatic or $dq = 0$) and the flow is essentially one dimensional ($r >> \ell$ and $dA/d\ell$ is continuous and small).

1. The fluid used for supersonic aerodynamic analysis is

_______________________________.

////////////////////////////////

homogeneous, compressible, non-viscous and elastic.

2. The basic equations are developed by assuming the control volume has the following properties:

a. Properties are not a function of time.

b. Adiabatic

c. Isentropic

d. One-dimensional

////////////////////////////////

a, b, d (isentropic process assumption is not required for basic equations)

Rigorously $a^2 \equiv \left.\frac{\partial P}{\partial \rho}\right)_{\Delta S = 0}$, but in an isentropic process, $P\rho^{-\gamma}$ = constant and simple mathematics gives $a^2 = \gamma \frac{P}{\rho}$ which combined with the ideal gas law, $P = \rho RT$, gives:

$a^2 = \gamma RT$ or

T is in degrees Rankine

$a = 49\sqrt{T}$ ft/sec (for air)

$a = 29\sqrt{T}$ Kts (for air)

3. The speed of sound is a function of ____________________.

////////////////////////////////

temperature only

Mach number is defined as a velocity divided by a speed of sound. Its physical significance can be considered to relate the Kinetic energy to the internal energy of a flow.

$$M^2 \sim \frac{V^2}{\gamma RT} \sim \frac{\text{Kinetic energy}}{\text{internal energy}}$$

Although we normally think of Mach number in terms of the local speed of sound, it can be and often is defined in other ways.

Based on local temperature

$$M = \frac{V}{a}$$

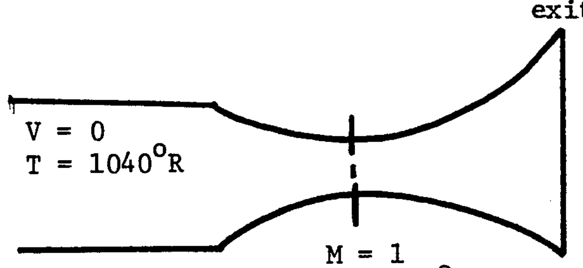
Based on stagnation temperature

$$M_T = \frac{V}{a_T}$$

Based on "Local sonic" temperature

$$M^* = \frac{V}{a^*}$$

4.



At the exit plane

a. M = __________

b. M_T = __________

c. M* = __________.

////////////////////////////////

$$M = \frac{V_{exit}}{49\sqrt{T}} = \frac{2000}{49\sqrt{510}} = 1.81$$

$$M_T = \frac{V_{exit}}{49\sqrt{T_T}} = \frac{2000}{49\sqrt{1040}} = 1.27$$

$$M^* = \frac{V_{exit}}{49\sqrt{T^*}} = \frac{2000}{49\sqrt{625}} = 1.63$$

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UNIT TWO

SOUND WAVE PROPAGATION

AND

TEMPERATURE PRESSURE AND DENSITY

RELATIONSHIPS

1. Sound waves are ______________ and propagate at ______________.

    a. pressure pulses; a speed which depends on the pressure.

    b. pressure pulses; acoustic velocity

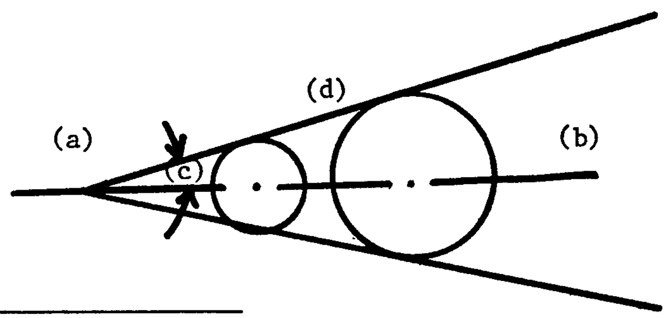
    c. not isentropic; acoustic velocity

    d. infinite; the speed of sound

////////////////////////////////

b.

2. Label the following diagram:



a. ________________

b. ________________

c. ________________

d. ________________

////////////////////////////////

a. Zone of silence

b. Zone of activity

c. μ (Mach angle) = arc sin $\frac{1}{M}$

d. Wave front or wave cone

3. The formulae $\frac{T_T}{T} = 1 + \frac{(\gamma - 1)}{M} M^2$ and $\frac{P_T}{P} = (\frac{T_T}{T})^{\gamma/(\gamma-1)}$ are good for ________________ and ________________ flow respectively.

a. adiabatic; adiabatic

b. isentropic; isentropic

c. adiabatic; isentropic

d. isentropic; adiabatic

/ / / / / / / / / / / / / /

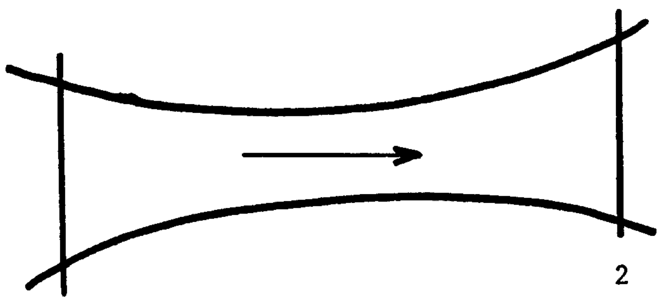
b,c

4. P_T, ρ_T and T_T are constant for ________________, ______________, and ________________ flow respectively.

////////////////////////////////

isentropic, isentropic, and adiabatic (or isentropic)

5.



2

1

$M_2 = 1.8$

$M_1 = 0.6$
$P_1 = 20\ lb/in^2$

For isentropic flow find

a. P_{T_1}

b. P_{T_2}

c. P_2

////////////////////////////////

a. $25.5\ lb/in^2$ b. $25.5\ lb/in^2$ c. $4.44\ lb/in^2$

If you correctly answered the above questions, you have completed Unit 2 Go to page 3-1.

If you think you need more instruction, complete the following pages.

TEACHING MATERIALS

Sound waves are a series of alternate compression and rarefaction pressure pulses transmitted in all directions at a speed proportional to the temperature of the fluid called acoustic velocity or more commonly, the speed of sound. If the source of the sound waves is moving, the sound waves do not remain concentric rings around a point but as the speed of the source equals or exceeds the speed of sound produces distinct zones as shown in Figure 2-1.

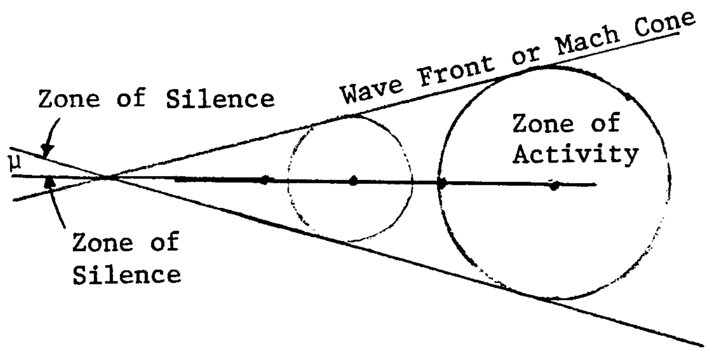


Figure 2-1

All effects produced by a body moving at supersonic speed are contained within the zone of activity. Conversely, any arbitrary point in a supersonic stream can only be affected by disturbance from points lying on or within a cone of the vertex angle μ extending upstream. The region outside the cone of activity is called the cone of silence.

1. Sound waves are ______________ and propagate at _________________.

   a. ______________________

   b. ______________________

////////////////

a. Pressure pulses

b. The speed of sound/acoustic velocity.

2. The zone of activity is ____

__________________________.

////////////////

The cone extending downstream from a pressure disturbance having a vertex angle μ in which all effects produced by the source of the disturbance are contained.

By use of the restricted energy equation which assumed steady, one dimensional, adiabatic flow and a calorically perfect gas, an equation for the ratio of total to static temperature as a function of Mach number can be developed.

$$\frac{T_T}{T} = 1 + \frac{(\gamma - 1)}{2} M^2$$

To develop the pressure and density ratios as a function of Mach number, a further restriction of isentropic flow is assumed. These formulas are:

$$\frac{P_T}{P} = \left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_T}{\rho} = \left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{1}{\gamma - 1}}$$

These ratios are tabulated as functions of Mach number in NACA Report 1135.

Since total quantities are constant for isentropic flow (and Total Temperature is constant for adiabatic flow), the static properties of pressure, temperature and density at two points in an isentropic flow are related to the respective Mach numbers.

$$\frac{P_1}{P_2} = \left(\frac{P_{T2}}{P_2}\right)\left(\frac{P_{T1}}{P_1}\right)^{-1} = f(M_2)\ f(M_1)$$

3. The static to total temperature ratio as a function of Mach number is good for ____________ flow, while the pressure and density ratios are restricted to ______________ flow.

a. ______________

b. ______________

////////////////

a. adiabatic

b. isentropic

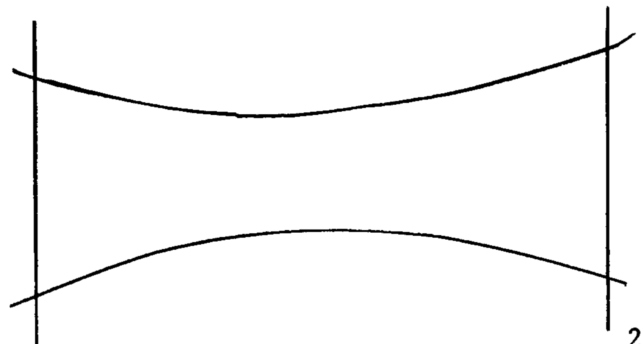
4. Total temperature, density and pressure are ______________ in isentropic flow.

a. ______________

////////////////

a. constant

5.



$M_1 = 0.8$ $M_2 = 1.3$

$T_1 = 400\ ^{\circ}R$

$P_1 = 25\ lb/in^2$

For adiabatic flow:

a. $T_{T1} =$

b. $T_2 =$

c. $P_2 =$

////////////////

a. $T_{T1} = \left[\left(T_1 \right) \left(\frac{T_{T1}}{T_1} \right) \right] = \frac{400\ ^{\circ}R}{0.8865} = 451\ ^{\circ}R$

b. $T_2 = \left[\left(\frac{T_{T2}}{T_2} \right) \left(\frac{T_{T1}}{T_{T2}} \right) T_{T1} \right]^{-1} = (0.7474)\ (1)\ (451) = 337\ ^{\circ}R$

c. Can not be determined at this point in course , because this flow was said to be adiabatic. If it were isentropic, you could assume P/P_T = constant and use the same method as in b. above.

UNIT THREE

CONVERGING/DIVERGING STREAMTUBE

AND

CHOKED FLOW

1. A nozzle ______________________________ and a diffuser __________

_________________.

   a. converges, diverges

   b. diverges, converges

   c. accelerates flow, decelerates flow

   d. decelerates flow, accelerates flow.

////////////////

c

2. A converging streamtube is a (a) ______________________________ in subsonic flow and a (b) ______________________ in supersonic flow.

   a. ______________________

   b. ______________________

////////////////

   a. Nozzle

   b. Diffuser,

3. If $dA = 0$ then Mach number must equal one.

   a. True

   b. False

////////////////

   b. False , it can equal one, but not must equal one.

4. When M = 1 at the throat, then the flow is ______________.

a. Supersonic downstream of the throat

b. Subsonic downstream of the throat

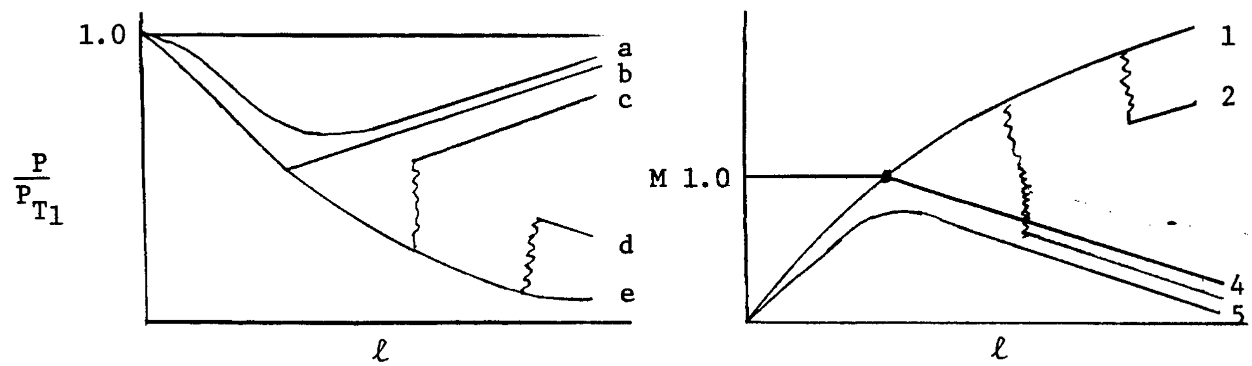
c. Can not occur because of shocks

d. Choked.

////////////////

d.

Refer to Figure 5-1 to answer questions 5-8 for a C-D streamtube.



5. Match up the pressure ratios with the Mach numbers.

a. ______________

b. ______________

c. ______________

d. ______________

e. ______________

////////////////

a. 5

b. 4

c. 3

d. Not shown (impossible)

e. 1

6. Case #2 is ______________________.

____

////////////////

impossible.

7. Cases (a)____________________ are isentropic and cases (b)______________ ____________ are adiabatic.

a. ______________________

b. ______________________

////////////////

a. a, b, e

b. a, b, c, e.

8. Cases __________________ have the same A/A* at the exit.

____

////////////////

a, b, e, (1, 4, 5)

9. To change mass flow rate during choked flow ________________________.

a. Increase exit pressure

b. Decrease inlet total pressure

c. Increase A_{throat}.

d. Increase Mach number at throat.

////////////////

b, c.

10. If exit pressure is less than ambient the nozzle is too ____________ ____________ and the flow is ________________________.

a. Short, underexpanded

b. Long, overexpanded

c. Short, overexpanded

d. Long, underexpanded.

////////////////

b.

If you correctly answered the above questions, you have completed Unit 3. Go to page 4-1

If you think you need more instruction, complete the following pages.

TEACHING MATERIALS

The definition of a nozzle and a diffuser are based on their action on a flow not on geometrical considerations. A converging $(dA/d\ell < 0)$ - diverging $(dA/d\ell > 0)$ streamtube will affect the flow differently in supersonic flow than in subsonic flow. The affect will be governed by the relationship:

$$\frac{dA}{A} = \frac{dV}{V} (M^2 - 1)$$

Since A and V are positive quantities, it can readily be seen that the relationship between dA and dV will be controlled by the sign of the expression $(M^2 - 1)$.

For subsonic flow $(M^2 - 1)$ is negative. So when dA is negative (convergent case) dV is positive and flow accelerates (nozzle). When dA is positive (divergent case), dV is negative and flow decelerates (diffuser).

For supersonic flow $(M^2 - 1)$ is positive. So when dA is negative (convergent case), dV is negative and flow decelerates (diffuser). When dA is positive (divergent case), dV is positive and flow accelerates (nozzle).

Thus a nozzle (converging streamtube) in subsonic flow reacts as a diffuser in supersonic flow and vice versa.

When $M = 1$, then $(M^2 - 1)$ is equal to zero and dA must equal zero. This is by definition at the throat of a converging-diverging (C-D) streamtube. Thus if $M = 1$ in a C-D streamtube, this can only occur at the throat ($dA = 0$). However the converse is not true, if $dA = 0$, the Mach number does not have to equal one; dV can also be zero and satisfy the equation with $M \neq 1$.

The case in a C-D streamtube where $M = 1$ at the throat has some unique properties and flow satisfying this condition is called CHOKED FLOW.

1. A nozzle (a)______________ flow and a diffuser (b)______________ flow.

////////////////

a. accelerates

b. decelerates

2. A diverging streamtube is a (a)____________ in subsonic flow and a (b)______________ in supersonic flow.

////////////////

a. diffuser

b. nozzle.

3. If $M = 1$ in a C-D streamtube then dA = ______________.

////////////////

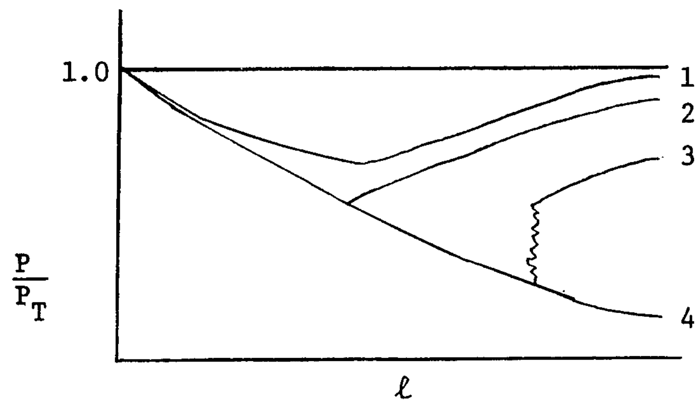
zero.

4. Choked flow occurs in a C-D streamtube when ______________/

////////////////

$M = 1$ at the throat.

If we start with a C-D streamtube with inlet and outlet ambient pressure equal, no flow occurs. If we then cause a decrease in the outlet ambient



pressure flow will occur and a pressure drop will be experienced as velocity increases in the convergent portion of the streamtube. However since flow is subsonic everywhere, the pressure increases as the velocity decreases in the divergent portion of the streamtube (case 1). As we continue to decrease the outlet ambient pressure, we will reach a point where $M = 1$ exactly at the throat resulting in choked flow. From this point further decreases in outlet ambient pressure will not have any affect on the flow upstream of the throat since flow is moving as fast as (or faster than) the effects of a pressure change can move through the flow (zone of silence). At this point (case 2) the outlet ambient pressure is not low enough to allow the flow to accelerate past the throat, dA, dV and (M^2-1) equal zero at the throat and the flow decelerates with pressure increasing in the divergent portion of the streamtube. If we decrease the outlet ambient pressure some more (case 3), the

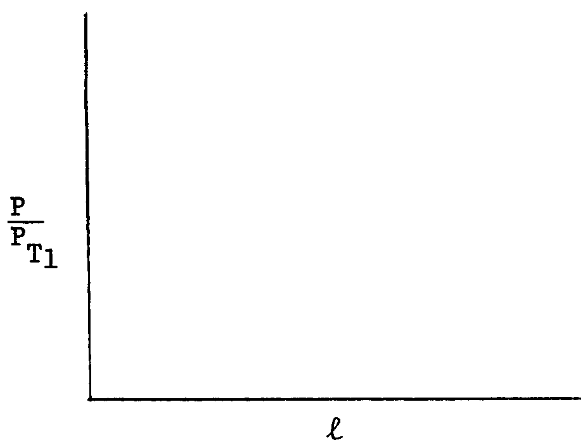
flow continues to accelerate past the throat and becomes supersonic with a corresponding pressure decrease initially. However for the exit pressure to equal the outlet ambient pressure, a flow discontinuity will occur. For outlet ambient pressures between Case 2 and Case 4, the flow discontinuity will start as a normal shock near the throat and the shock will move towards the exit as the outlet ambient pressure is decreased. Pressure reductions past this point prior to Case 4 will result in a series of oblique shock outside of the nozzle. When the outlet ambient pressure is low enough (Case 4) no shocks occur and C-D streamtube is an ideal nozzle.

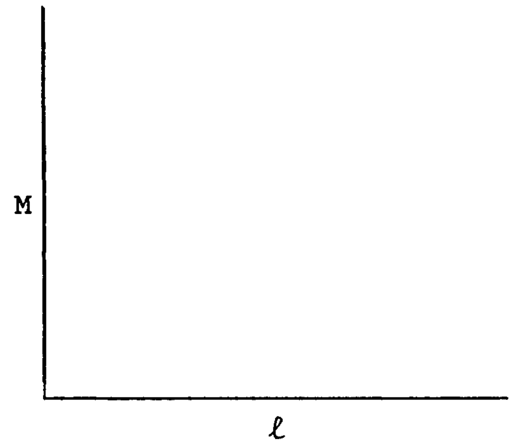
For outlet ambient pressures between Case 2 and Case 4, the flow is overexpanded and the nozzle is too long. For outlet ambient pressures below Case 4, the flow is underexpanded and the nozzle is too short.

When a shock occurs during the flow, that part of the flow is adiabatic; for the rest of the flow we assume isentropic flow.

By setting $\frac{\partial \dot{m}}{\partial M} = 0$ we can find the maximum mass flow rate. The maximum mass flow rate for given entrance conditions occurs when $M = 1$ (or for choked flow). To increase max mass flow rate either entry conditions or geometry must be changed.

1. Draw the pressure ratio and Mach number changes through a C-D nozzle for (a) isentropic choked flow, subsonic in divergent section, (b) choked flow normal shock, (c) ideal nozzle.





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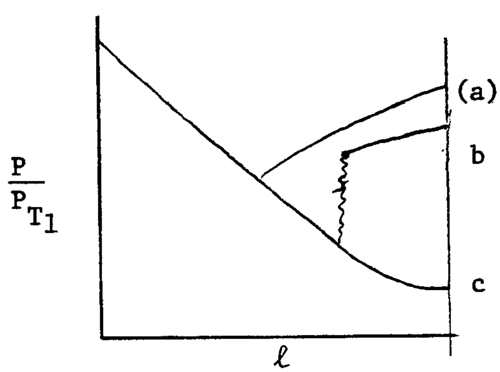

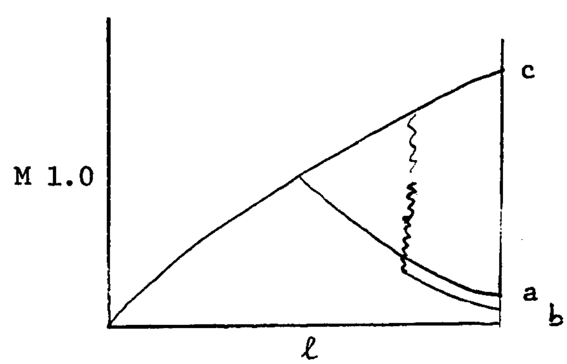

2. Reference question 1, case(s) (a)______________ are isentropic,

case(s) (b)__________________ are adabatic.

a. ____________________

b. ____________________

////////////////

a. a, c

b. a, b, c.

3. Maximum massflow rate always occurs

a. When ρAV = constant

b. When M = 1 at throat

c. During choked flow

d. If a shock occurs in or beyond the exit duct

////////////////

b, c, d.

UNIT FOUR

LOCAL SONIC PROPERTIES

AND

NORMAL SHOCKS

1. Local Sonic Properties labeled $( )^*$ are defined only:

   a. Where $M^* = 1$

   b. Where $M = 1$

   c. As those that would exist if $M^* = 1$ (isentropically)

   d. As those that would exist if $M = 1$ (isentropically).

////////////////

d

2. Local Sonic Properties have a constant relationship with Total Properties.

   a. True

   b. False

////////////////

a

3. A^* is constant for:

   a. Adiabatic flow

   b. Isentropic flow

   c. Transonic flow

   d. None of the above.

////////////////

b

4. M^* is:

   a. V^*/a^*

   b. Bounded

   c. Equal to one when $M = 1$

   d. Greater than one when $M > 1$

////////////////

   b, c, d.

5. A/A^*

   a. Is constant for isentropic flow

   b. Is equal to one for choked flow

   c. Will determine the Mach number in the flow

   d. Will determine one of two possible Mach numbers in isentropic flow.

////////////////

   d (except $M = 1$)

6. A normal shock is:

   a. Adiabatic

   b. Isentropic

   c. "Visible" region of flow discontinuity

   d. Perpendicular to flow direction.

////////////////

   a, c, d.

7.

$M_1 = 1.5$

$T_1 = 590\ ^\circ R$

$P_{T1} = 15$ PSI

FIND:

M_2 =

T_2 =

T_{T2} =

P_{T2} =

////////////////

M_2 = .7011

T_2 = (1.320) (590 °R) = 779 °R

T_{T2} = T_{T1} = 590/.6897 = 855 °R

P_{T2} = (0.9298)(15 PSI) = 13.95 PSI

If you correctly answered the above questions you have completed Unit 4
Go to page 6-1

If you think you need more instruction complete the following pages.

Local sonic properties labeled with a superscript *, are those properties that would exist if the flow conditions were changed to $M = 1$ isentropically. The pressure, temperature, and density total properties are constant in an isentropic flow. By use of the relationships between static and total properties as a function of Mach number, and using $M = 1$, the following equations can be found for $\gamma = 1.4$.

$$P^* = 0.528 \ P_T$$

$$\rho^* = 0.634 \ \rho_T$$

$$T^* = 0.833 \ T_T$$

Thus if T^* is known (like knowing the velocity at the throat of a choked nozzle, $V = a^* = \sqrt{\gamma R T^*}$,) then the total temperature can be found.

The quantity A^* becomes very valuable in solving numerous flow problems. This quantity is the area you would have to expand or contract the flow to get $M = 1$. From our definition of choked flow it can be seen that $A_{TH}\big|_{M=1} = A^*$.

A^* can be shown to be constant for an isentropic flow, but will change during adabiatic only flow.

M^* is different than other * quantities. It is defined by the equation $M^* = V/a^*$ since if it were defined like the other * quantities the answer would be trivial, i.e. $M^* \equiv \frac{V^*}{a^*} = 1$, this is not true! M^* can be found as a function of M.

$$M^{*2} = \frac{\gamma + 1}{2/M^2 + \gamma - 1}$$

If $M < 1$ then $M^* < 1$

If $M = 1$ then $M^* = 1$

If $M > 1$ then $M^* > 1$

If $M \to \infty$ then $M^* \to \sqrt{6}$ for $\gamma = 1.4$

NACA 1135 lists M^* that corresponds to M and if you know V and a^* at a point, you can locate the corresponding M and thus all the associated ratios in the NACA 1135 tables.

The ratio A/A^* can be used to find the Mach number and associated ratios of static and total properties, however for each A/A^* value there are two Mach numbers, one subsonic and one supersonic that satisfy the equation for A/A^* as a function of M. Thus to resolve the ambiguity additional information is required.

1. Local sonic properties are defined as:

   ____.

   ////////////////

   Those that would exist if flow changed to $M = 1$ isentropically.

2. A^* is constant for ____.

   ////////////////

   isentropic flow.

3. M^* is defined as ____.

   ////////////////

   V/a^*

4. The A/A^* for $M = 0.5$ is the same as for $M =$ ________________.

////////////////

$M = 1.7$

A normal shock is a flow region in which large changes in fluid properties take place within a very short flow distance. We assume that the flow is adiabatic ($dq = 0$ since a parcel of air remains in the shock wave only a short time) and that the cross-sectional area of streamtube is unchanged across a shock wave. The shock wave is perpendicular to the flow direction and this region of discontinuity can be made visible (width $\approx 10^{-5}$ inches).

With a large dose of mathematics the ratios of static properties across a shock and the downstream Mach number can be found as a function of the upstream Mach number. These ratios are tabulated in most aerodynamics books for air ($\gamma = 1.4$) and can be found in NACA 1135.

To use the NACA 1135 tables let's look at an example problem:

$M_1 = 1.3$ M_2

$P_1 = 10$ psi

$T_1 = 600$ °R

To find M_2 look in the 11th column (page 22) labeled M_2 for $M_1 = 1.3$ and find $M_2 = .7860$. To find P_2 look in the 12th column labeled P_2/P_1 and find $P_2/P_1 = 1.805$. P_2 then equals (1.805)(10 PSI) or 18.05 PSI to find T_{T_2} look in the 4th column labeled T/T_T and find $T/T_T = .7474$. T_{T_1} then equals (600)/(.7474) or 803 °R since $T_{T_2} = T_{T_1}$ then $T_{T_2} = 803$ °R.

Figure 4-2 shows how some typical properties change across a normal shock.

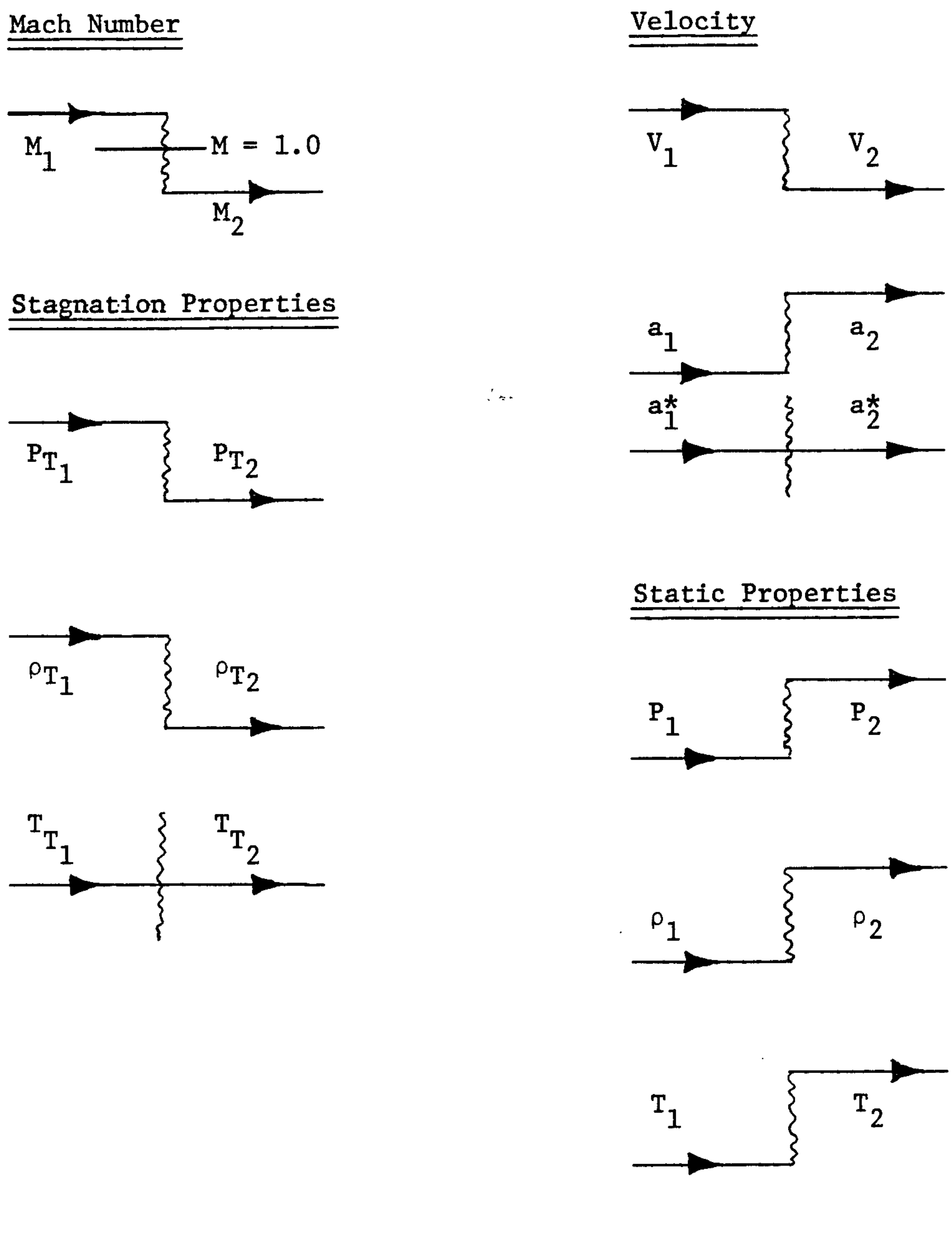

Figure 4-2

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UNIT SIX

OBLIQUE SHOCKS

1. The wave angle (θ) is

   a. The angle the flow turns through across an oblique shock

   b. The angle the oblique shock makes with the upstream streamlines

   c. Has two values for each upstream Mach number

   d. $\mu \leq \theta \leq \pi/2$

////////////////

b, c, d.

2. The turning angle (δ) is

   a. The wedge angle

   b. The angle the flow turns through across an oblique shock

   c. The angle the oblique shock makes with the upstream streamlines

   d. $\mu \leq \delta \leq \pi/2$

////////////////

a, b.

3. When $\delta > \delta_{max}$ for a given Mach number

   a. Detachment occurs

   b. The shock becomes a Mach line

   c. An attached normal shock occurs

   d. A zero strength shock occurs.

////////////////

a.

4. For $M = 2.0$, the minimum wave angle is (a) ______________________ and the maximum wave angle is (b) ______________________.

a. ________________

b. ________________

////////////////

a. 30^{o}

b. 90^{o}

5.

The wave angle (weak shock wave) is (a) ______________. The maximum turning angle for an attached shock wave is (b) __________. The ratio of P_2/P_1 is (c) ____________.

a. __________

b. __________

c. __________

////////////////

a. 39.3^{o}

b. 23^{o}

c. 1.715.

If you correctly answered the above questions you have completed Unit 6. Go to page 7-1.

If you think you need more instruction complete the following pages.

TEACHING MATERIALS

In nature the normal shock is an anomaly, the oblique shock is the more "normal" case. The additional degree of freedom requires additional information. For a normal shock the up stream Mach number was sufficient. For an oblique shock the Mach number and an angle are required. The number of degrees the flow must turn due to a concave corner is called the turning angle or wedge angle (δ). The angle the oblique shock makes with the upstream streamlines is called the wave angle (θ), Figure 6-1.

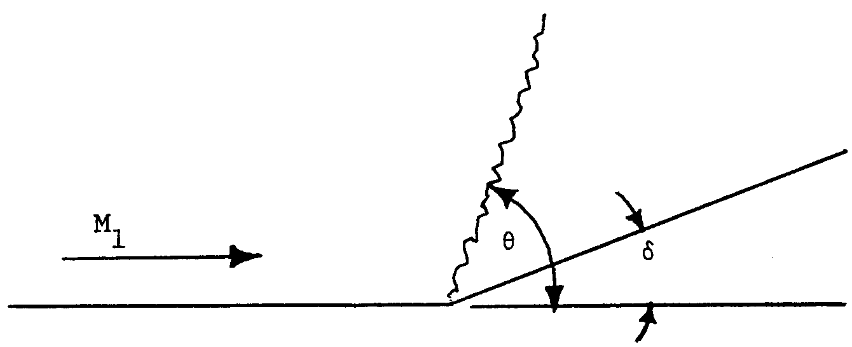


FIGURE 6-1

For a shock wave to occur, $M_1 \sin\theta \geq 1$.

For the case $M_1 \sin\theta = 1$, then $\sin\theta = \frac{1}{M_1}$ and $\theta = \mu$, the Mach line and we have a zero strength shock. The normal shock is the maximum limit and $\mu \leq \theta \leq \pi/2$.

1. The wave angle (θ) is ____

____________________.

////////////////

The angle the oblique shock makes with the upstream streamlines.

2. The turning angle (δ) is ____

____________________.

////////////////

The angle the flow turns through across an oblique shock.

A relationship for M_1, δ and θ can be found after much mathematical manipulation.

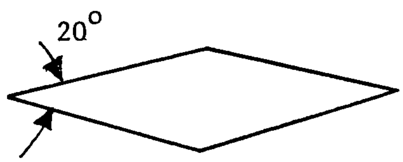
$$\tan \delta = 2 \operatorname{ctn} \theta \frac{M_1^2 \sin^2 \theta - 1}{M_1^2 (\gamma + \cos 2\theta) + 2}$$

This can be represented as a graph such as found on page 42 of the NACA 1135 tables. Examination of this graph shows several interesting points. First for each Mach number there is a maximum turning angle, beyond which the shock wave separates from the corner. Second for each value of δ and M_1 there are two values of θ. The lower value of θ is called the weak solution and is the one most likely to occur.

The ratio of properties across a shock (static to static or total to total, but not static to total) can be found in NACA 1135 tables by using

the normal component of the up-stream Mach number. For example for $\delta = 8^{o}$ and $M_1 = 2.5$ then $\theta = 30^{o}$ and $M_{1n} = 1.25$. Use Mach number of 1.25 to enter the NACA tables to find the ratio of properties of interest.

1. The diamond wing shown is flown at an AOA of 5^{o} at a Mach number of 2.3. The wave angle is (a) ________________. The normal Mach number is (b) ________________. The static pressure on the forward top side of the wing at sea level is (c) ________________.



a. ________________

b. ________________

c. ________________

////////////////

(a) 30^{o}

(b) 1.15

(c) 20.2 PSI

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C-v

UNIT SEVEN

ISENTROPIC EXPANSION

1. $\Delta\nu$ is ____

____.

////////////////

The angle a supersonic flow is turned away from the upstream streamline.

2. In an isentropic expansion, the activity (property changes) take place in a fan defined by the angles ν_1 and ν_2.

   a. True

   b. False

////////////////

b.

3. Boundary condition for the Prandtl-Meyer function are:

   a. $\nu = 0$ when $M = 0$

   b. $\nu = 0$ when $M = 1$

   c. $\nu = \pi/2$ when $M = 0$

   d. $\nu = \pi/2$ when $M = 1$

////////////////

b.

4. Supersonic flow has accelerated around a corner and has reached $M_2 = 2.5$. If the flow approaching the corner was at $M_1 = 1.2$, the turning angle of the corner is (a) ____________________.

The value of P_2/P_1 is (b) ____________________.

a. ____________

b. ____________

////////////////

a. 35.6°

b. 0.1419

If you correctly answered the above questions you have completed Unit 7, go to page 8-1.

If you think you need more instruction complete the following pages.

We have seen that when a supersonic flow is turned into itself, a change in flow properties takes place. This change takes place in a small region called a shock and produces a compression and a reduction in velocity. But it can be shown by entropy considerations that this shock wave effect does not occur when flow is turned away from itself. An expansion, which is isentropic, takes place across a broad region (expansion fan) determined by the incoming Mach number and the angle of flow change. The angle a supersonic flow is turned away from the upstream streamlines is defined as $\Delta\nu$. The expansion fan is defined by the angles μ_1, and μ_2, the wave angles corresponding to the upstream Mach number (M_1) and the downstream Mach number (M_2). Figure 7-1.

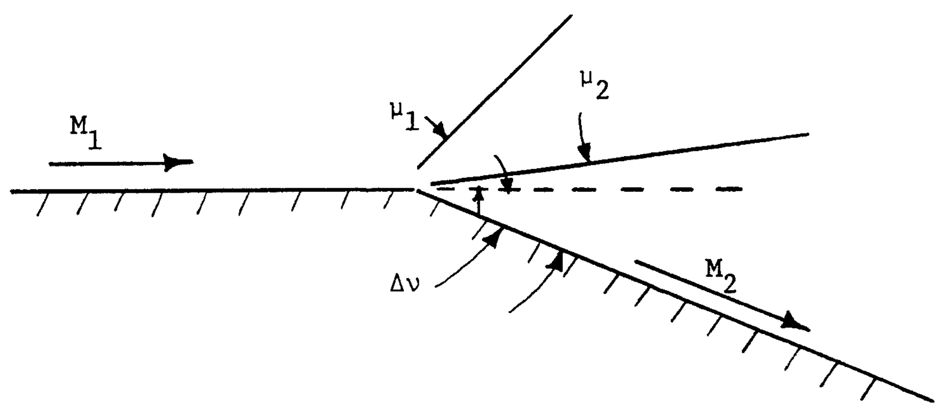

Figure 7-1

A relationship between the Mach number and the turning angle can be found called the Prandtl-Meyer function:

$$\nu = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M}$$

The boundary conditions for this function are chosen such that $\nu = 0$ when $M = 1$. Thus physically ν_1 is the angle the flow is turned to accelerate it from $M = 1.0$ to M_1 and $\Delta\nu = \nu_2 - \nu_1$. Thus if M_1 and $\Delta\nu$ are known, then ν_2 can be calculated by $\nu_2 = \Delta\nu + \nu_1$. And the value of M_2 can be found from tables like NACA 1135 by looking up ν_2 and finding its associated Mach number. For example if $M_1 = 2.0$ and $\Delta\nu = 15^\circ$ then look up ν_1 on page 22 column 9 and find 26.38°. Thus $\nu_2 = 41.38^\circ$ and go down column 9 until you find $\nu = 41.415$ (close enough for government work) and find $M_2 = 2.6$. Since an expansion is isentropic the total properties are constant, i.e. $P_{T_1} = P_{T_2}$, $T_{T_1} = T_{T_2}$, $\rho_{T_1} = \rho_{T_2}$. Thus the static properties can be found by use of the static to total ratios for the two Mach numbers.

For small compression turns the Prandtl-Meyer function can also be used with good accuracy by just using $\Delta\nu$ as a negative number, i.e., for $\delta = 5^\circ$ then use $\Delta\nu = -5^\circ$.

1. ν_1 is ____

____.

////////////////

The angle a flow would have to turn through to accelerate from $M = 1.0$ to M_1.

2. The Prandtl-Meyer function is valid for (a) ____________ flow

(a) ______________

////////////////

isentropic

3. Supersonic flow at $M_1 = 2.3$ approaches a corner of 10^o away from the flow. The downstream Mach number (M_2) is (a) ____________. The static density ratio ρ_2/ρ_1 is (b) ______________.

(a) ____________

(b) ____________

////////////////

a. 2.73

b. .62

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UNIT EIGHT

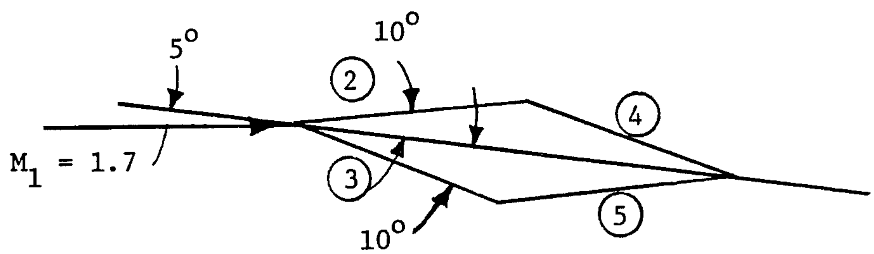
PRESSURE DISTRIBUTION

ABOUT A WING

AND

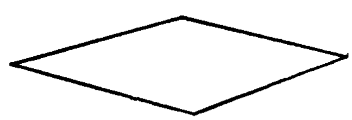
THIN WING THEORY

1. Given the symmetrical airfoil below. The airfoil is at an angle of attack 5^{o} and the flow has a Mach number of 1.7.



(a) Draw the appropriate shocks and expansion fans. The pressure coefficient in region 2 is (b)________, in region 3 is (c)__________, in region 4 is (d)__________ and in region 5 is (e)____________.
C_L for the wing is (f)__________ and C_D for the wing is (g)________.

a.



b.

c.

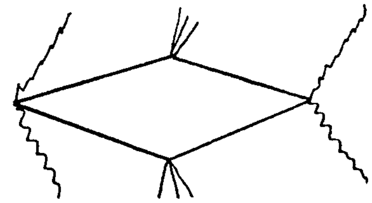
d.

e.

f.

g.

////////////////

a. 

b. 0.14

c. 0.57

d. -0.280

e. -0.112

f. 0.290

g. 0.123

2. Using thin wing theory the C_L for the wing in the previous problem is (a)____________ and the C_D for the wing is (b)____________.

a____________

b____________

////////////////

a. 0.252

b. 0.112

3. Ackeret (thin wing) theory agrees well with experimental data from Mach numbers of about 1.2 to 5.0.

a. True

b. False

////////////////

a.

4. Three dimensional supersonic wings do not ever have tip losses.

a. True

b. False

////////////////

b.

5. A subsonic airfoil section should not be used for a supersonic aircraft.

   a. True

   b. False

////////////////

b.

If you correctly answered the above questions you have completed Unit 8, go to page 9-1.

If you think you need more instruction complete the following pages.

Previous lessons have provided the tools necessary to define the lift and drag forces (or coefficients) on simple two dimensional wings. A simple wing can be divided into compressive (oblique shock) regions and expansive (Prandtl-Meyer) regions. (Figure 8-1) From the definition of pressure coefficient

$$C_p \equiv \frac{P - P_\infty}{q_\infty}$$

can be derived the local pressure coefficient on an airfoil as

$$C_p\Big)_x \equiv \frac{P_x - P_\infty}{q_\infty}$$

or in terms of the flow Mach number

$$C_p\Big)_x = \frac{2}{M_\infty^2 \gamma}\left(\frac{P_x}{P_\infty} - 1\right)$$

For a double wedge airfoil (Figure 8-1)

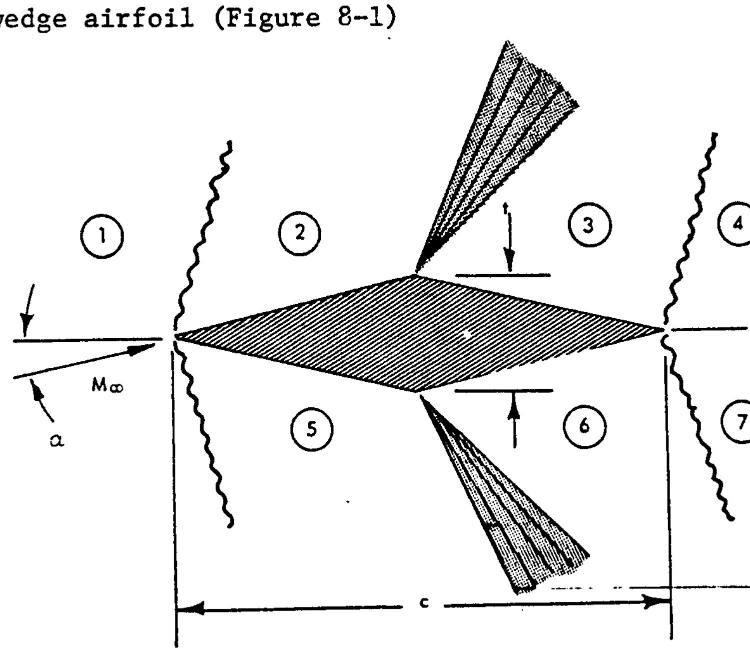


Figure 8-1

The pressure ratio $\frac{P_6}{P_\infty}$ can be found as

$$\frac{P_6}{P_\infty} = \left(\frac{P_6}{P_{T_6}}\right)\left(\frac{P_{T_6}}{P_{T_5}}\right)\left(\frac{P_{T_5}}{P_5}\right)\left(\frac{P_5}{P_\infty}\right)$$

$$\frac{P_6}{P_{T_6}} = f(M_6)$$

$$\frac{P_{T_6}}{P_{T_5}} = 1$$

$$\frac{P_{T_5}}{P_5} = f(M_5)$$

$$\frac{P_5}{P_\infty} = f\left(M_{\infty_n}\right)$$

By use of geometry and NACA 1135 Tables each of the Mach numbers can be found with the appropriate oblique shock or Prandtl-Meyer expansion relationships. Then the pressures and pressure coefficients can be calculated. The normal forces can be found by:

$$F_x = \left|C_{p_x}\right| q_\infty S$$

And the lift and drag forces are found by resolving the force components into forces parallel and perpendicular to the relative wind.

This can be a very tedious process and approximate solutions have been found. One of these approximate solutions widely accepted is Ackeret (or Linear) Theory. For small angles of attack, the Ackeret Theory produces acceptable results from 1.2 to 5.0 Mach numbers. Linear Theory predicts that:

$$C_p = \pm \frac{2\,\delta}{\sqrt{M^2 - 1}}$$

where the minus sign holds for an expansion, the plus sign holds for a compression and δ is the angle through which the flow is turned.

For a double wedge the theory predicts that:

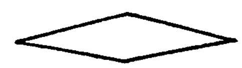
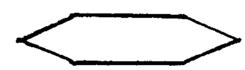
$$C_L = \frac{4\,\alpha}{\sqrt{M^2 - 1}}$$

$$C_D = \frac{4}{\sqrt{M^2 - 1}} \left(\frac{t}{c}\right)^2 + \frac{4\,\alpha^2}{\sqrt{M^2 - 1}}$$

The second term in the C_D equation can be thought of as the drag due to lift and is all that is left if we assume that $\left(\frac{t}{c}\right)$ becomes very small (flat plate). The first term can be considered as profile drag due to wing shape and takes the more general form:

$$C_{D_p} = \frac{K}{\sqrt{M^2 - 1}} \left(\frac{t}{c}\right)$$

The values of K for different shapes are listed in Table 8-1

Shape	K	
Flat Plate	0	
Double Wedge	4	
Biconvex	5.33	
Modified Double Wedge	2/ b*	

* b is the fraction of the chord length, of the wedge shape, at one end.

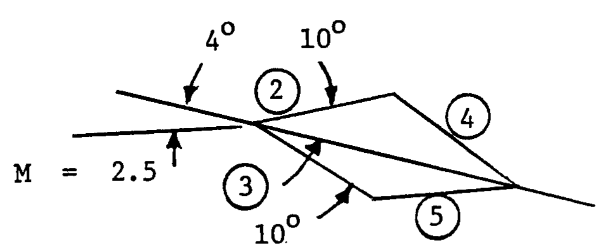
Table 8-1 implies that the double-wedge type of cross-section is the optimum shape since it has the least drag for a given thickness ratio (neglecting the flat plate as non-practical). However other things need be considered like strength and manufacturing ease. A shape may have higher drag for a given thickness ratio but be stronger than a lower drag shape.

The biconvex type may be considered the optimum practical shape to give the least drag per unit stress. When stress is not the limiting criterion, then the double wedge would be advantageous because of its low drag per unit thickness. From a practical standpoint, the modified double wedge is simpler to machine, when solid sections are required, and has only slightly increased drag per unit stress.

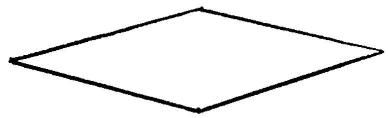
Thus far only two dimensional wings have been considered. Each point on a wing will generate a cone of disturbance. When the leading edge of a wing is behind the Mach cone then the normal Mach number is subsonic and no shock wave is created at the leading edge. The pressure distribution will be equivalent to those found on an airfoil normal to the stream at the corresponding subsonic Mach number. If the wing will always be below the effective Mach number of one then it is advantageous to use a subsonic airfoil. If the effective Mach number is greater than one, the Mach cone is behind the wing and a supersonic airfoil must be used.

Tip losses are also involved in three dimensional wings. These losses are confined to the region within the tip cones. If the wing is cut off so that none of the wing is within the Mach cone the wing will act like a two-dimensional wing and have no induced effects (tip losses).

1. The symmetrical wing shown is at an angle of attack of 4^{o} and a Mach number of 2.5



(a) Draw the appropriate shocks and expansion fans. The pressure coefficient in region 2 is (b) ________. in region 3 is (c) ________. in region 4 is (d) ________ and in region 5 is (e) ________. C_L for the wing is (f) ________ and C_D for the wing is (g) ________.

a. 

b.

c.

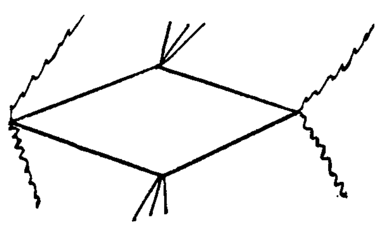
d.

e.

f.

g.

///////////////

a. 

b. 0.105

c. 0.3

d. -0.146

e. -0.0765

f. 0.135

g. 0.055

2. Using Linear Theory C_L for the wing in the previous example is (a) ________________. C_D is (b) ____________________.

a. ________________

b. ________________

////////////////

a. 0.122

b. 0.0628

UNIT NINE

TRANSONIC FLOW

PRETEST

1. Transonic flow range extends from (a) ______________________ to (b) ______________________.

a.

b.

////////////////

a. when sonic flow first occurs over the surface of the vehicle (Mach Crit)

b. when flow is supersonic over the entire vehicle.

2. Increasing the sweep angle of a wing

a. Decreases drag

b. Raises the wing critical Mach number

c. Lowers the wing critical Mach number

d. Increases lift.

////////////////

b.

3. An aircraft in the transonic region is very stable due to the existence of an almost normal shock on the vehicle.

a. True

b. False

////////////////

b.

4. Drag during transonic flow can be altered by:

a.

b.

c.

d.

etc.

////////////////

a. sweeping wings

b. transonic area rule

c. changing aspect ratio

d. super critical wing

etc.

If you correctly answered the above questions you have completed Unit 9.

Review all course material then go to page 10-1 to complete the one hour practice test.

If you think you need instruction complete the following pages.

Transonic flow is a highly complex problem which defies direct mathematical analysis. Even experimental results from a wind tunnel are difficult to obtain due to the choking effects when a model is placed in the throat of a tunnel at near sonic conditions. We will only look at some qualitative aspects of the transonic flow range. This range begins when flow over the vehicle reaches a sonic velocity somewhere on the surface. This freestream speed is the critical Mach number of the vehicle and is always less than one. As the freestream velocity increases, the region of supersonic flow over the vehicle increases. The supersonic flow region is always bounded aft by a shock. As the velocity increases further, a bow wave develops forward of the leading edge of the vehicle. At higher Mach numbers, the bow wave closely approaches the leading edge, depending upon the shape of the leading edge. For a sharp edge, the bow wave may become attached to the leading edge. At this point the vehicle has passed out of the transonic region and is supersonic, with the flow supersonic over the entire vehicle.

Shortly after an airfoil reaches its critical Mach number, a significant drag increase is noted and lift decreases due to formation of the shock wave and separation of the boundary layer due to shock wave/boundary layer interaction. Thus the aerodynamist is very interested in increasing the critical Mach number and thus delaying the drag rise. Several tools are available to do this. A primary tool is the use of sweepback. The aerodynamic characteristics of a wing are determined by the velocity component perpendicular to

the wing axis. This implies that the critical Mach number of a wing with sweepback is increased over one without sweepback according to:

$$M_{CR} = \frac{M'_{CR}}{\cos \Lambda}$$

M_{CR} is critical Mach number of wing with sweepback Λ

M'_{CR} is critical Mach number of wing without sweepback.

The actual increase in critical Mach number is never as great as predicted by the above formula because of three-dimensional effects. Besides delaying the drag rise, wing sweep reduces the magnitude of the drag rise and allows use of a subsonic airfoil in some cases. However, the penalities include a reduction in the lift curve slope, increased drag at higher Mach numbers and reduced effectiveness of trailing edge devices.

Aspect ratio will also affect critical Mach number. However this effect is very small until below an aspect ratio of three to four.

Another method of transonic drag reduction, attributed to Dr. Whitcomb, is the area rule. Dr. Whitcomb found that by contracting the fuselage in the area of the wing, such that the cross-sectional area of the wing fuselage perpendicular to the fuselage was reduced and smoothed when plotted against the fuselage length, the drag was reduced in the transonic area. This design effect can be seen in the F-105 and the T-38.

Dr. Whitcomb is also credited with major work in the development of wing profiles which shift the drag-critical Mach numbers to higher values by reducing the profile thickness ratio. Through suitable profile design, local areas of supersonic flow can be created on the profile in which recompression to subsonic flow occurs steadily or in weak shock waves only. On these profiles the pressure rise in the recompression zone is gradual and thus does not cause flow separation. These wings are termed supercritical wings.

Two effects of transonic/supersonic flight that caused significant problems for the designer, tester and user are center of pressure shift and boundary layer separation. It can be shown that the lift on an airfoil at subsonic speed acts at 25% MAC and shifts to 50% MAC at supersonic speeds. This causes the Mach Tuck or dig that you all have experienced during your 4 G decells from supersonic flight in the F-4. The magnitude of this effect is related to the amount of movement of the center of pressure compared to the distance between the C.G. and center of pressure.

The boundary layer separation problem is related to the pressure gradient along the surface. The rapid pressure increase through the shock increases this pressure gradient and causes the boundary layer to separate aft of the shock. This separation is an erratic process, which does lead to some control problems.

It also is responsible for a pressure drag increase and turbulence in the separated flow can cause severe airframe buffet.

1. The critical Mach number is ______________________________
____.

////////////////

The freestream Mach number where sonic flow first appears on the airfoil surface.

2. Three ways that drag during transonic flight can be altered are
(a) ____
(b) ____
(c) ____.

////////////////

a. sweeping wings

b. area rule

c. super critical wing

d. changing aspect ratio.

UNIT TEN

PRACTICE

SUPERSONIC AERODYNAMICS

EXAM

	POINTS
1.	10
2.	6
3.	10
4.	10
5.	3
6.	6
7.	10
8.	15
9.	30
	100

(10) 1. Read the question and circle the correct answer:

T F Adiabatic flow is isentropic and reversible.

T F In a convergent-divergent streamtube the maximum mass flow occurs only at the throat.

T F a* is constant across an oblique shock.

T F During an expansion supersonic flow accelerates and static density increases.

T F Choked flow in a convergent-divergent streamtube has a maximum mass flow only when it is ideally expanded.

T F Flow prior to the throat in a convergent-divergent nozzle is always subsonic.

T F The Ackeret Thin wing theory agrees well with experimental data from Mach numbers of about 1.2 to 5.0.

T F Static temperature increases through an isentropic expansion.

T F The Mach number behind an oblique shock is always supersonic.

T F Total density (stagnation density) decreases behind an oblique shock.

(6) 2. Given an ideally expanded nozzle with supersonic flow at the exit and an exit pressure ratio of .1011

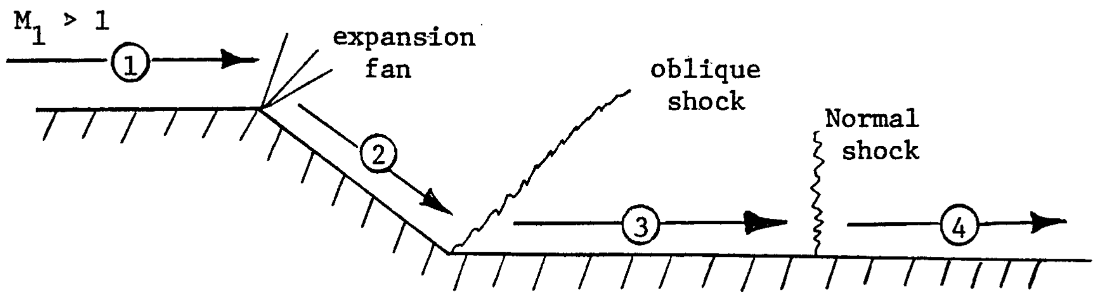
(1) a. Is the Mach number prior to the throat supersonic or subsonic?

(1) b. What is the exit Mach number?

(3) c. To what pressure ratio would you change the exit pressure ratio to maintain choked flow and have subsonic flow at the exit without any shocks?

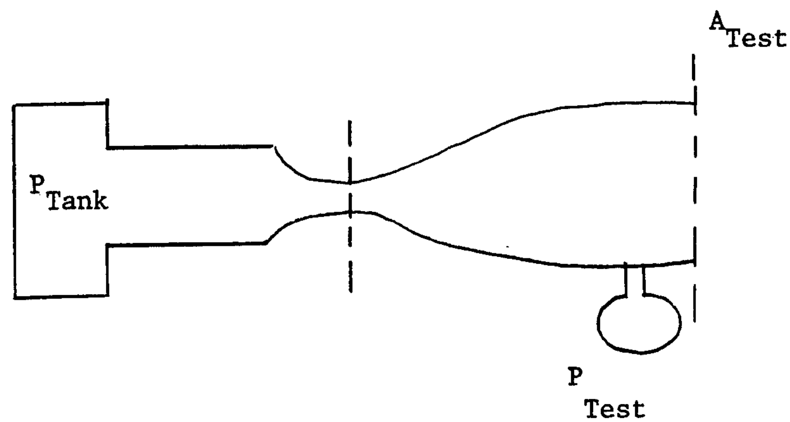
(1) d. What will be the exit Mach number after part (c) is accomplished?

(10) 3. For the following compressible nonviscous flow situation existing in part of a duct, complete the following relationships using the notation >, <, = , and ? for unknown.



T_{T_4}	T_{T_3}	M_4	1
P_{T_3}	P_{T_2}	M_4^*	M_3^*
P_4	P_2		
a_3	a_2		
M_3	1		
ρ_{T_2}	ρ_{T_1}		
a_2	a_1		
A_4^*	A_3^*		

(10) 4. Given the wind tunnel below, $P_{Tank} = 1000\ lb/ft^2$ and $A_{Test} = 2.5\ A^*$. During operation the pressure in the test section was measured to be $63.27\ lb/ft^2$.



(4) a. Find the Mach number in the test section?

(4) b. Find the test section Mach number if P_{Test} goes to 5.6 PSI ?

(2) c. Was your pressure gauge reading correctly during the second pressure reading? Assume isentropic and nonviscous flow.

(3) 5. Name three ways to reduce transonic drag (improve transonic performance).

(6) 6. Identify 6 assumptions made during development of the $\left(\frac{T_T}{T}\right)$ equation.

(10) 7. Define by words or symbols:

M^*

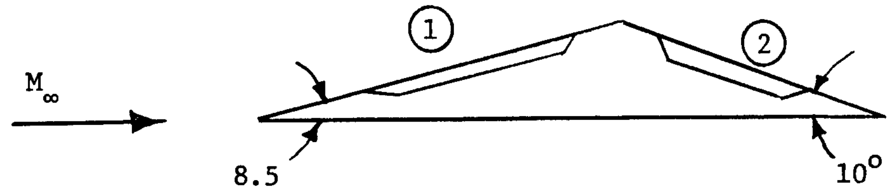
T_T

μ

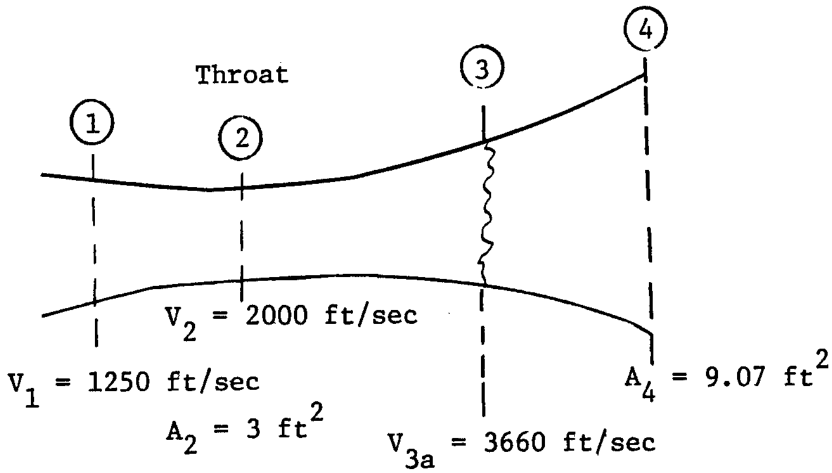
A^*

ν

(15) 8. The F-15 SPO has requested that you fly the WHITLY-6 camera pod to photograph a supersonic release of the F-15 drop tanks at 2.3 Mach. You are naturally concerned about the camera pods structural integrity at higher Mach numbers. To maintain a margin of safety you decide to compute the local Mach number and static pressure in regions 1 and 2 of the lens wedge profile as shown below for $M_\infty = 2.5$ and $p_\infty = 785 \text{ lb/ft}^2$. Sketch the shock and expansion patterns formed.



(30) 9. Given the following streamtube with a normal shock located as shown



(4) a. What is Mach number downstream of normal shock (station 3b)?

(4) b. What is stagnation temperature at station 1 ?

(2) c. What is stagnation temperature at station 4 ?

(8) 9d. What is static temperature at station (4) ?

(4) 9e. What is area at station (1) ?

(8) 9f. If we shortened the streamtube to attain an ideally expanded flow what would be the exit area? Draw its location on the streamtube above.